Table of Contents

Compiled by Thomas L. Grimes

New: topics listed here are bookmarked.

June 2009

Algebra - An Introduction to key definitions and concepts

PREFACE

I spent many hours developing of this study guide. I have attempted to make it (1) useful to learn or re-learn concepts and skills, but (2) more importantly, to be comprehensive reference guide to review and study mathematics concepts and skills. It includes concepts, procedures, and details often omitted or condensed in middle school/junior high school texts.

Major topics here are not always presented in a developmental order (the order that you might expect to learn them), but, more than in most texts, topics are presented more completely (reviewing old concepts and definitions before introducing new ones) and new concepts are integrated with old concepts in a manner to make it clear how the old and new concepts affect each other.

I hope this guide makes your life easier.

Symbols and abbreviations	Meanings and descriptions
+	1, add, addition; 2, positive
–	1, subtract, subtraction; 2, negative
x, ∗, •, ( )	multiply, multiplication: 2x3=2 ∗3= 2• 3= (2)3=2(3)=(2)(3)=6
÷, /	divide, division: 6 ÷ 3 = 6/3 = 2
=	equals, is
≠	not equal to, is not
≈	approximately equal
>	greater than
<	less than
≥	greater than or equal to
≤	less than or equal to
±	plus or minus. Example ± 5 means the range of -5 to +5
5^2	5 raised to the second power = 5X5 = 5 squared = 25
5^3	5 raised to the third power = 5X5X5 = 5 cubed = 125
sq ft	square feet or feet squared example: 7 sq ft = 7 ft2
GCF; LCF	Greatest common factor; Least common factor
LCM; LCD	Least common multiple; Least Common Denominator
5:7	the ratio 5 to 7; 5/7
...	repeats forever
P (x, y)	point P with coordinates x, y
^	is perpendicular to
||	is parallel to
∆ABC	triangle ABC
/____\ M N	line MN (arrows on the ends indicate the line continues indefinitely)
$\overline{AB}$ A B	(line) segment AB
Other concepts	where B represents any value.
/B/	the value of B
|B|	the absolute value of B
B^3	B raised to the third power = B x B x B = B^3 = B cubed
–B	the opposite of B; negative B

Algebra - An Introduction to key definitions and concepts

The (n) and (v) listed immediately in defined math words, concepts, and principals herein denote noun and verb and have no mathematical meaning.

algebra - (n) a method of solving practical problems by using symbols, usually letters, for unknown quantities; an advanced arithmetic in which symbols, usually letters of the alphabet, represent numbers or members of a specified set of numbers and are related by operations that hold for all numbers in the set.

operations - (n) the mathematical operations are adding, subtracting, multiplying, and dividing

algebraic - (adjective) of, about, or pertaining to algebra; the adjective form of the noun algebra.

properties - (noun) in mathematics, properties list the qualities (characteristics) of the members of a group, set, or class. A property defines mathematical nature of a group, set, or class of numbers and in turn imposes the rules, procedures, and processes to accurately identify or calculate those members. To arrive at the correct answers and solutions, you must understand the properties (content, concepts, implications, and applications) of the numbers and/or members with which you are working.

Naming or associating a particular property with its content or concept is not nearly as important as actually understanding the content and concepts of a property; however, standard tests and book publishers' tests often ask you to do just that. I, too, sometimes have difficulty recalling the name of a particular property or associate a name with its concepts. My advice is to "go with the flow." Review the names of properties before tests.

developmental (order) - (adjective) information is arranged in an order that aids learning and understanding concepts and skills being presented for the first time. This is what texts do. This guide is not a substitute for your textbook. Work most the textbook problems (with given answers) in each assigned set until you get the correct answer and you will do fine.

systematized (order) - (adjective) arranged in an order that shows how each piece (concept, principal, theory, etc.) fits into the whole scheme; in mathematics, concepts, numbers, or members are lumped together because they possess the same or similar attributes and they must be manipulated in the same or slightly different manner. This is what you must know to internalize (correctly relate new concepts to old concepts) to understand, perform, and remember your math. This guide was written to aid you in that task.

Information In this study guide is more systematized than textbooks usually are; hopefully it will help you to put the pieces together. Be warned, the pieces (concepts, principals, theories, etc.) come vey fast here, but you will not have to wade through pages and pages of your textbook to find how to work a problem.

This guide is used across grades (6 through 8), so brothers, sisters, friends, and neighbors in other grades may now use it or have used it. They may be willing and able help you; they don't have to look in an unfamiliar textbook to show you where to begin.

A quick arithmetic review:

sum - (n) the answer when adding

difference - (n) the answer when subtracting

product - (n) the answer when multiplying

quotient - (n) the answer when dividing

Algebra - An Introduction to key definitions and concepts

DEFINITIONS and CONCEPTS on this and the next page are listed in developmental order. (Developmental order is defined on the previous page.)

algebra - (n) a method of solving practical problems by using symbols, usually letters, for unknown quantities; a advanced arithmetic in which symbols, usually letters of the alphabet, represent numbers or members of a specified set of numbers and are related by operations that hold for all numbers in the set.

constant - (n) a value that stays the same such as the value a number; 8 always has the same value; π is always equal to 3.14; π is pronounced Pi

variable- (n) a symbol, usually a letter, used to represent a number

operation - (n) (mathematical operation) add, subtract, multiply and divide are the mathematical operations represented by symbols +, -, X, and ÷

term - (n) in the equation, 2X + 3Y - 25 = 0 , the terms are 2X, 3Y, 25, AND 0. Terms are usually separated by addition and/or subtraction operations symbols (+, -) and the equals symbol (=) Terms can be separated by < , > , ≤ , ≥ as well as =, +, AND -

like terms - (n) like terms (1) contain the same variable (letter) or variables and (2) variables are raised to the same power; 2X and 5X are like terms; 5Xy and 3Xy are like terms; 8b and $12b^2$ are unlike terms because "b" is not raised to the same power. Power is defined and described at the top of the next page.

unlike terms - (n) In the expression, 6x and 3y, the terms are unlike because the variables (X and Y) are not the same. 6x and $6x^2$ are unlike terms because the variable x is not raised to the same power.

expression - (n) (mathematical expression) - an expression represents a number; it is statement of at least two terms connected by one or more operations symbols +, -, X, and/or ÷. An example of an expression is 2X + 3y - 25 See page 5 for a list of examples.

polynomial - (n) an expression of one or more terms; examples: 3x + y and X + 2xy + y are polynomials.

binomial - (n) a polynomial with two terms. Example: 2x + 3y

monomial - (n) a "polynomial" with one term. Example: 5x

sentence - (n) a mathematical sentence contains either = , < , > , ≤ , ≥

equation - (n) a mathematical sentence that contains an equals symbol =

base and exponent - (nouns) In 3^4, 3 is the base, and 4 is the exponent. The exponent tells how many times the base is multiplied by itself to attain a regular (natural) number.

Example $3^4 = 3 * 3 * 3 * 3 = 81$

power - (noun) 3^4 means 3 raised to the fourth power or the fourth power of 3; same as example immediately above. In this sense, "power" and "exponent" mean the same thing.

coefficient - (n) the number 5 is the coefficient of X in the term 5X; a coefficient is a constant.

factor - (n) in multiplication, each number multiplied is a factor of the product. Then a number is a factor of a given number if the given number is divisible by the number. Example: 2 and 3 are factors of 6.

multiply – (v) to multiply, to perform multiplication.

multiple - (n) a product of a given number and a whole number. The multiples of 3 are 3, 6, 9, 12, and so on.

simplify - (v) to simplify an expression means to replace it with the least complicated equivalent expression

Example: 5X - 3X + 7 is simplified to 2X + 7

rearrange - (verb) to rearrange, in an expression like 9X + 8 - 12X + 7, we rearrange the terms to simplify:

9X + 8 - 12X + 7 is rearranged as (9X - 12X) + (8+7) to simplify to -3X + 15

combine - (v) to combine, like terms can be combined; unlike terms cannot be combined. See "like terms" and "unlike terms"

Example: $9X^2 + X + 3 + 4X + 5$ can be combined to $9X^2 + 5X + 8$

evaluate - (v) to find the value of an expression means to find its value when numbers are substituted for variables (letters) in the expression

What is the value of (X - 3), if X =7

If you substitute 7 for X, then (7 - 3) = 4

solve - (v) to solve an equation is to find all its solutions, the values of each variable (variables are usually represented by letters)

satisfy - (v) if a variable in an equation is replaced with a number, and the resulting statement is true, then the number satisfies the equation

Example :	Solve for X when	Satisfy (verify) that X is 16
	X - 7 = 9	X - 7 = 9
	X - 7 + 7 = 9 + 7	16 - 7 = 9
	X = 16	9 = 9

Algebra - An Introduction to key definitions and concepts

KEY WORDS & PHASES to set up problems

A mathematical word or mathematical expression, or word phase can represent a number. Key words indicate which mathematical operation(s) (add, subtract, multiply, and/or divide) to use to write mathematical expressions for word expressions.

ADD - Words & phrases that indicate ADD:

word expression	mathematical expression
the sum of N and b	N + b
N plus b	N + b
N exceeded by b	N + b
N increased by 3	N + 3
3 longer (heavier, wider, more ...) than N	N + 3

How many in all?
Total

SUBTRACT - Words & phrases that indicate SUBTRACT:

word expression	mathematical expression
the difference between X and 4	X - 4
X minus 4	X - 4
4 less than X	X - 4
X decreased by 4	X - 4
4 shorter (lighter, more narrow) than X	X - 4

How many more?
How many less?
Change? How much change?

MULTIPLY - Words & phrases that indicate MULTIPLY:

word expression	mathematical expression
the product of 5 and a	5 X a, 5·a, 5•a or 5a
5 times a	5a
5 multiplied by a	5a

NOTE: Refer to "PREFACE & SYMBOLS" (p-1) for symbols such as "•"

NOTE: 3 times 5 = 3 X 5 = 3•5 = 3 (5) = (3) 5 = (3)(5)

DIVIDE - Words & phrases that indicate DIVIDE:

word expression	mathematical expression
the quotient of 2 into X	X/2 or X ÷ 2
the quotient X divided by 2	X/2 or X ÷ 2
X divided 2	X/2 or X ÷ 2
How many 2's in X	X/2 or X ÷ 2

See the next page for examples and a review of these concepts

KEY WORDS & PHASES to set up problems, continued

word expression	mathematical expression
The sum of a number and 7	N + 7
Seven more than a number	N + 7
A number increased by 7	N + 7
The difference of a number and 7	N - 7
Seven less than a number	N - 7
A number decreased by 7	N - 7
The opposite of a number decreased by 7	-N - 7
The sum of twice a number and 7	2N + 7
The difference of twice a number and 7	2N - 7
The product of twice a number and 7	2N(7)
The sum of twice a number and -7	2N + (-7)
Five times the sum of twice a number and -7	5[2N + (-7)]
Five times the quantity of twice a number and negative seven	5[2N + (-7)]
Six times the sum of twice the opposite of a number and -7	6[2(-N) + (-7)]
The product of 3 and the sum of a number and 7	3(N + 7)
The sum of a number and 7, multiplied by 3	3(N + 7)
Three times the quantity of a number and 7	3(N + 7)
Five times the sum of a number and -7	5[N + (-7)]
Five times the difference of a number and 7	5(N - 7)

Algebraic properties: systematized order

PROPERTIES OF OPERATIONS (add, subtract, multiply & divide) IN EQUATIONS: FIRST LESSON

Addition property of equations - adding the same value to each side of an equation does not change the solution. ("c" is added to both sides in the equation below)

If a = b, then a + c = b + c

Subtraction property of equations - subtracting the same value "c" from each side of an equation does not change the solution; this property is not listed in some texts because subtracting a number is the same as adding its opposite.

("c" is subtracted from both sides in the equation below)

If a = b, then a - c = b - c

Multiplication property of equations - multiplying each side of an equation by the same value (number) does not change the solution.

If a = b, then a(c) = b(c) *otherwise written as* **ac = bc**

Division property of equations - dividing each side of an equation by the same number (except 0, zero) does not change the solution.

if a = b, then $\frac{a}{c} = \frac{b}{c}$

PROPERTIES OF OPERATIONS: ADVANCED LESSON - A BROADER VIEW

note: ab = a(b) = a x b	For any numbers a, b, and c where c ≠ 0	
Addition	For Equalities (equations)	For Inequalities
Subtraction	If a = b, then a + c = b + c	If a > b, then a + c > b + c. If a < b, then a + c < b + c.
Multiplication	If a = b, then a - c = b - c	If a > b, then a - c > b - c. If a < b, then a - c < b - c.
Division	If a = b, then ac = bc	See the next page.

Algebraic properties: systematized order, continued

PROPERTIES OF OPERATIONS:	ADVANCED LESSON - A BROADER VIEW
	For Equalities (equations)
Addition Property	For any numbers a, b, and c: If a = b, then a + c = b + c
Subtraction Property	For any numbers a, b, and c: If a = b, then a - c = b - c
Multiplication Property	For any numbers a, b, and c If a = b, then ac = bc
Division Property	For any numbers a, b, and c where c ≠ 0 If a = b, then $\frac{a}{c} = \frac{b}{c}$
	For Inequalities
Addition Property	For any numbers a, b, and c: If a > b, then a + c > b + c. If a < b, then a + c < b + c.
Subtraction Property	For any numbers a, b, and c: If a > b, then a - c > b - c. If a < b, then a - c < b - c.
Multiplication Property	For any numbers a, b, and c: If a > b and c > 0, then ac > bc If a > b and c < 0, then ac < bc
Division Property	For any numbers a, b, and c: If a > b and c > 0, then $\frac{a}{c} > \frac{b}{c}$ If a > b and c < 0, then $\frac{a}{c} < \frac{b}{c}$
Trichotomy Property	For all real numbers a and b, one and only one of the following is true: a = b a < b a > b

If c < 0 then c is negative. If when solving inequalities you multiply by or divide by a negative number, the inequality symbol switches to the opposite inequality symbol: from > to <, or from < to >, or from ≥ to ≤, or from ≥ to ≤

Examples:

$-3a > 6$	$\frac{-3a}{-3} < \frac{6}{-3}$ dividing by -3	$a < -2$	$\frac{a}{-3} < 2$	$\left(\frac{-3}{1}\right)\left(\frac{a}{-3}\right) > (-3)(2)$ multiplying by -3	$a > -6$

Whatever operation (add, subtract, etc.) that is performed on one side of an equation (or inequality), must also be performed on the other side of that equation (or inequality).

Algebraic Properties: systematized list, continued

REAL NUMBER PROPERTIES	For every a, b, and c that are real numbers	
ab, a • b, a(b), a x b, and a ∗ b are all ways to express a times b	Of Addition (Additive)	Of Multiplication (Multiplicative)
Associative Property; <u>regroup</u> ##	(a + b)+ c = a + (b + c) example (4 + 5) + 3 = 4 + (5 + 3) 9 + 3 = 4 + 8 12 = 12 regrouping a, b, and c does not change their sum; also the addition of (a+b+c) can be done in any order	(ab) c = a (bc) example (4•5) •3 = 4• (5•3) 20•3 = 4•15 60 = 60 regrouping a, b, and c does not change their product; also the multiplication of (a∗b∗c) can be done in any order
Commutative Property; <u>reorder</u> ##	a + b = b + a example 3 + 2 = 3 + 2 proof 5 = 5	a • b = b • a example 3 • 2 = 3 • 2 proof 6 = 6
Closure Property	the sum of a + b is a unique real number	the product of ab is a unique real number
Identity Property	a + 0 = 0 + a = a	a • 1 = 1 • a = a
(Additive) Inverse Property.	a + (-a) = 0	$a \bullet \frac{1}{a} = 1$; $a\left(\frac{1}{a}\right) = 1$
Property of (-1)		(-1)a = a(-1) = -a
Property of 0 (zero)		a • 0 = 0; 0 • a = 0
Distributive Property	a(b + c) = ab + ac	
Reflexive Property	a= a (A number is equal to itself.)	
Symmetrical Property	If a = b, then b = a.	
Transitive Property	If a = b and b = c, then a = c.	
Substitution Property	If a = b, then a can be replaced by b and b by a.	

While there is no associative or commutative property for either subtraction or division, consider that dividing by 5 is the same as multiplying by 1/5, and subtracting a number is the same as adding its opposite.

Algebraic Properties: Systematized List, Continued

Property of Exponents

For Multiplication example $a^3 \bullet a^4 = a^{3+4} = a^7$	**For all real numbers a, and for positive integers m and n:** $a^m \bullet a^n = a^{m+n}$
For Division examples $\frac{a^6}{a^2} = a^{6-2} = a^4 = a \bullet a \bullet a \bullet a$ $\frac{a^2}{a^6} = a^{2-6} = a^{-4} = \frac{1}{a^4}$ $\frac{a^4}{a^4} = a^{4-4} = a^0 = 1$	**For all real numbers a, a ≠ 0, and for positive integers m and n:** If m > n, then $\frac{a^m}{a^n} = a^{m-n}$ If m < n, then $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$ If m = n, then $\frac{a^m}{a^n} = a^{m-n} = a^0 = 1$

Product Property of Square Roots

example $\sqrt{20} = \sqrt{4} \bullet \sqrt{5}$	For all real numbers m and n where m ≥ 0 and n ≥ 0 , $\sqrt{mn} = \sqrt{m} \bullet \sqrt{n}$

Review item

(Additive) Inverse Property.	a + (-a) = 0	$a \bullet \frac{1}{a} = 1$; $a\left(\frac{1}{a}\right) = 1$

inverse (n) 1. (general use) something that is opposite in character; the reverse.
2. (mathematics) the reciprocal of a quantity (also called multiplicative inverse) or the negative of a quantity (also called additive inverse.)

By other names

ab = a • b = a(b) = a x b = a∗b are other ways to express a times b 4a = 4 • a = 4(a) = (4)(a) are other ways to express 4 times a	a is the opposite of (-a) is another way to say a is the additive inverse of (-a) Note that a + (-a) = 0	$\frac{1}{a}$ is the reciprocal of a is another way to say $\frac{1}{a}$ is multiplicative inverse of a Note that $a \bullet \frac{1}{a} = 1$; $a\left(\frac{1}{a}\right) = 1$

ALGEBRAIC PROPERTIES, an alphabetized select order (See "properties" on page 3.)

Name of Property (or concept)	Application
addition property of equalities for any numbers a, b, and c	If a = b, then a + c = b + c
addition property of inequalities for any numbers a, b, and c	If a > b, then a + c > b + c. If a < b, then a + c < b + c.
associative property of addition for any numbers a, b , and c	(a + b) + c = a + (b + c) Therefore, addition can be done in any order.
associative property of multiplication for any numbers a, b , and c	(a • b) • c = a • (b • c)
closure property of addition for all rational numbers a and b	a + b is a rational number
closure property of multiplication for all rational numbers a and b	a • b is a rational number
commutative property of addition for any numbers a and b	a + b = b + a
commutative property of multiplication for any numbers a and b	a • b = b • a
distributive property for any numbers a, b, and c	a(b + c) = ab + ac; (b + c)a = ab + ac
division property for any numbers a, b, and c & c ≠ 0	if a = b, then $\frac{a}{c} = \frac{b}{c}$
identity property of addition for any number a	a + 0 = a
identity property of multiplication for any number a	a • 1 = a
inverse property of addition for every rational number a	a + (⁻a) = 0
inverse property of multiplication for every nonzero number $\frac{a}{b}$	where a ≠ 0, b ≠ 0 there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} * \frac{b}{a} = 1$
multiplication property of equality for any numbers a, b, and c	If a = b, then ac = bc
multiplicative inverse: defined reciprocal = multiplicative inverse $\frac{a}{b} * \frac{b}{a} = 1$	A number times its multiplicative inverse is equal to 1. Reciprocal is same as the multiplicative inverse.
multiplicative property of zero for any number a	a • 0 = 0; 0 • a = 0
opposite (inverse property of addition) the opposite of a is (⁻a)	⁺a + (⁻a) = 0
product property of square roots for any nonnegative numbers a and b	$\sqrt{a * b} = \sqrt{a} * \sqrt{b}$
proportions, property of	If $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$ If $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$
substitution property of equality for all numbers a and b	If a = b, then "a" may be replaced by "b"
subtraction property of equalities for any numbers a, b, and c	If a = b, then a - c = b - c

Algebra - An Introduction to key definitions and concepts

OPERATIONS (+,-,*, ÷) WITH MIXED (like & unlike) (positive & negative) SIGNS

Definitions and assumptions:

- Mathematical OPERATIONS are addition, subtraction, multiplication, and division.
- Mixed signs imply that both positive and negative numbers (integers) are present.
- If the sign of a number (integer or term) is not given, it is assumed to be positive.
- (+) represents a positive number; (-) represents a negative number

MULTIPLYING and DIVIDING integers with Mixed (Like & Unlike) Signs

When multiplying or dividing numbers with the same (like) signs, the answer is POSITIVE.

When multiplying or dividing numbers with different (unlike) signs, the answer is NEGATIVE.

MULTIPLICATION * = times

Multiplication of two integers (numbers) with like signs yields a positive product.

(+) * (+) = (+)

(-) * (-) = (+)

Multiplication of two integers with unlike (different) signs yields a negative product.

(+) * (-) = (-)

(-) * (+) = (-)

DIVISION / = divided by

Division of two numbers with like (same) signs gives a positive quotient.

(+) ÷ (+) = (+)

(+)/(+) = (+)

example (+4)/(+2) = (+8)

(-) ÷ (-) = (+)

(-)/(-) = (+)

example (-8)/(-2) = (+4)

Multiplication of two numbers with unlike (different) signs gives a negative quotient.

(+) ÷ (-) = (-)

(+)/(-) = (-)

(+8)/(-2) = (-4)

(-) ÷ (+) = (-)

(-)/(+) = (-)

(-8)/(+2) = (-4)

ADDING and SUBTRACTING with Mixed (Like & Unlike) Signs

ADDITION of two integers (numbers) with like (the same) signs: both integers (numbers) are either positive or negative,

The sum (answer) of two positive numbers is positive.

(+) + (+) = (+)

example (+8) + (+2) = (+10)

The sum (answer) of two negative numbers is negative.

(-) + (-) = (-)

(-8) + (-2) = (-10)

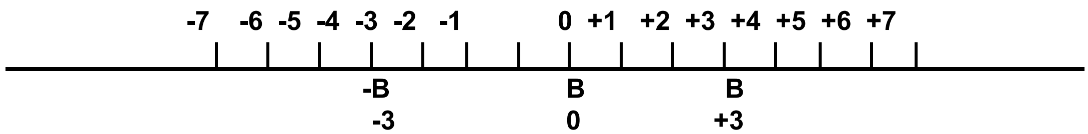
ADDITION of two integers (numbers) with unlike (different) signs.
In other words, one number is positive and the other is negative.

RULE: "WHEN ADDING POSITIVE AND NEGATIVE NUMBERS" (stated in more correctly) "WHEN COMBINING TERMS WITH UNLIKE SIGNS," you must SUBTRACT the smaller integer (number) from the larger integer (number), and the ANSWER TAKES THE SIGN of the larger integer (number or term). The sum may be positive, negative, or zero.

OPERATIONS (+,-,*, ÷) WITH MIXED SIGNS, continued

SUBTRACTING numbers (integers) with unlike (different) signs.

<u>Concept # 1</u>:
For every number there is an opposite number. On a number line, a number and its opposite are equal distance from zero. For example, in the diagram below, -3 and +3 are equal distances from zero.



The opposite of 5 is -5; the opposite of +a is –a; and the opposite of y is -y. The "+" sign may be given as in "+a" or implied as in "y".

NOTE: The property of additive inverse states that the sum of a number and its opposite are equal to 0. Example: +a + -a = 0

<u>Concept # 2</u>: You get the same answer whether you subtract a number or add its opposite.

Subtract a number	$^{+}7 - {}^{+}2 = {}^{+}5$	$^{-}3 - {}^{+}2 = {}^{-}5$
or add its opposite	$^{+}7 + {}^{-}2 = {}^{+}5$	$^{-}3 + {}^{-}2 = {}^{-}5$
Subtract a number	$^{+}6 - {}^{-}2 = {}^{+}8$	$^{-}3 - {}^{-}4 = {}^{+}1$
or add its opposite	$^{+}6 + {}^{+}2 = {}^{+}8$	$^{-}3 + {}^{+}4 = {}^{+}1$

RULE: TO SUBTRACT A NUMBER, ADD ITS OPPOSITE, and apply the rules of adding on the previous page.

One way to visualize (understand) all those signs (positive and negative numbers) and operation symbols (such as + for add and - for subtract) is to use parenthesis.

Subtract a number	$(^{+}7) - (^{+}2) = (^{+}5)$	$(^{-}3) - (^{+}2) = (^{-}5)$
or add its opposite	$(^{+}7) + (^{-}2) = (^{+}5)$	$(^{-}3) + (^{-}2) = (^{-}5)$
Subtract a number	$(^{+}6) - (^{-}2) = (^{+}8)$	$(^{-}3) - (^{-}4) = (^{+}1)$
or add its opposite	$(^{+}6) + (^{+}2) = (^{+}8)$	$(^{-}3) + (^{+}4) = (^{+}1)$

ORDER OF OPERATIONS: <u>one-step</u> solutions (to calculate unknown variables)

• The rules 1-3 to evaluate/simplify expressions at the bottom of the page also apply when solving equations for unknowns. •• Any operation that is performed on one side of the equation (left or right of the equals marks), must also be performed on the other side of the equation.

See "properties of operations in equations: first lesson" on page 7.

Solving Multiple-step Equations is on the next page.

Examples of One-step solutions

If 3 is added to the unknown A, then subtract 3 from both sides of the equation to solve for A. A + 3 = 7 A + 3 - 3 = 7 - 3 A = 4	If 3 is subtracted from unknown A, then add 3 to both sides of the equation to solve for A. A - 3 = 7 A - 3+ 3 = 7 + 3 A = 10
If 3 is multiplied by the unknown A, then divide both sides of the equation by 3 to solve for A. 3A = 12 3A/3 = 12/3 A = 4 Note: 3A = 3 times A = (3)(A)= 3xA = 3•A = 3 ∗A	If unknown A is divided 3, then multiply both sides of the equation by 3 to solve for A. A/3 = 7 3 • A/3 = 3 • 7 A = 21 Note: A/3 = A ÷ 3 and 3 ∗7 = 3 times 7

Concepts:

If something "S" is added to the unknown A, then subtract that something "S" from both sides of the equation to solve for A. A + S = 7 A + S - S = 7 - S A = 7 - S	If something "S" is subtracted from unknown A, then add that something "S" to both sides of the equation to solve for A. A – S = 7 A – S + S = 7 + S A = 7 + S
If something "S" is multiplied by the unknown A, then divide both sides of the equation by something "S" to solve for A. SA = 12 SA/S = 12/S A = 12/S Note: SA = S times A = (S)(A)= SxA = S•A = S ∗A	If unknown A is divided something "S", then multiply both sides of the equation by that something "S" to solve for A. A/S = 7 S ∗ A/S = S ∗ 7 A = 7S Note: A/S = A ÷ S

<u>Evaluating or simplifying expressions</u>:

Summary: First do what's in ( ), then second do the X and ÷, and finally do the + and –

First, accomplish (do) operations in parentheses; also, accomplish any operations above or below division bars before performing other operations.

$$\frac{2(5+4)}{6}+1=\frac{2(9)}{6}+1=\frac{18}{6}+1=3+1=4$$

Second, accomplish all multiplications and divisions in order from left to right.

Third and finally, accomplish all additions and subtractions in order from left to right.

CORRECT: 2 X 3 + 4 = ?
6 + 4 = 10

WRONG: 2 X 3 + 4 = ?
2 X 7 = 14
2 X 3 + 4 ≠ 14

Inside the parentheses, accomplish all multiplications and divisions and then all additions and subtractions in order from left to right.

ORDER OF OPERATIONS: multiple-step solutions (for unknowns/variables)
• The rules 1-3 to evaluate/simplify expressions at the bottom of the page also apply when solving equations for unknowns. •• Any operation that is performed on one side of the equation (left or right of the equals marks), must also be performed on the other side of the equation. See "Properties of operations in equations is on page 7.
Solving one-step equations is on the previous page.

To solve multiple-step equations, steps **1, 2,** and **3** must be performed in the given order.

1 If the unknown "A" appears on both sides of the equation, then you must perform operations as necessary to collect (get) all terms containing the unknown A" on the one side of the equation or other (your choice) without violating rules. Your choice should be based upon (1) seeking a positive "A" term (rather than negative) and/or the ease and simplicity of calculations.

7A - 5 = 3A
7A - 3A - 5 = 3A - 3A Collect all terms containing "A" on one side;
4A - 5 = 0 then combine terms containing "A" together.

2 Isolate the term containing the unknown from other term(s) as follows:

If something (3 in this example) is added to the term containing the unknown "A", then subtract 3 from both sides of the equation. A + 3 = 7 A + 3 - 3 = 7 - 3 A = 4	If something (5 in this example) is subtracted from the term containing the unknown "A", then add 5 to both sides. A - 5 = 7 A - 5 + 5 = 7 + 5 A = 12

3 Isolate the unknown "A" from other elements in the term containing the unknown "A". If the term containing the unknown has, in addition the unknown A, another quantity, then divide (or multiply) both sides of the equation by that quantity to solve for the unknown "A".

If something (5 in this example) is multiplied by the unknown "A", then divide both sides of the equation by 5 to solve for "A". 5A = 20 5A/5 = 20/5 A = 4 Note: 5A = 5 times A = (5)(A)= 5xA = 5 • A =5 ∗ A	If unknown "A" is divided by something (4 in this example), then multiply both sides of the equation by 4 to solve for "A". A/4 = 7 4 • A/4 = 4 • 7 A = 28 Note: A/4 = A ÷ 4 and 4 • 7 = 4 times 7

SUMMARY: When evaluating or simplifying expressions:

First do what's in ( ), then second do the X and ÷, and finally do the + and –

First, accomplish (do) operations in parentheses; also, accomplish any operations above or below division bars before performing other operations.

$$\frac{2(5+4)}{6}+1=\frac{2(9)}{6}+1=\frac{18}{6}+1=3+1=4$$

Second, accomplish all multiplications and divisions in order from left to right.

Third and finally, accomplish all additions and subtractions in order from left to right.

CORRECT: 2 X 3 + 4 = ? 6 + 4 = 10	WRONG: 2 X 3 + 4 = ? 2 X 7 = 14 2 X 3 + 4 ≠ 14

Inside the parentheses, accomplish all multiplications and divisions and then all additions and subtractions in order from left to right.

numbers - (noun) are mental constructs (i.e., our inventions, our symbols) to describe
(1) how many objects are in a group. (i.e., the size of a group),
(2) the size of an individual object (how big or how small something is), and
(3) an item's position (rank or standing) among a group of items.

NOTE: M times N can be expressed as MN, M•N, M(N), M x N, and M✱N
2 times 3 can be expressed as 2•3, 2(3), 2 x 3, and 2✱3
2 times b can be expressed as 2N, 2•N, 2(N), 2 x N, and 2✱N, but 2N is preferred;
it is an expected practice in algebra to express 2 times N as 2N
coefficient – the number 2 is also a coefficient of "N" in the expression 2N;
coefficients are constants (never change in amount or value)

The (n) and (v) listed immediately in defined math words, concepts, and principals herein denote noun and verb and have no mathematical meaning.

standard - (adjective) means the normal, ordinary, common, routine, every-day

notation – (n) (in mathematics) the way to show a certain value, amount, or quantity

standard numeral or standard notation - (n) the normal, ordinary way we denote (indicate, show) a certain value, amount or quantity: nine as 9; forty-nine as 49; and two hundred sixteen as 216.

superscript – (n) a number or symbol placed just to the right and above another numeral or symbol. The "2" in 6^2, the "3" in 10^3, the "n" in 10^n are superscripts.

subscript – (n) a number or symbol placed to the right and below another numeral or symbol. The "2" in R_2, the "3" in I_3, the "n" in D_n are subscripts.

base and exponent - (nouns) In 6^3, "6" is the base, and "3" is the exponent. The exponent is a superscript; it is a number or a symbol placed to the upper right side of another numeral or symbol called the "base" to indicate the power to which base is to be raised or how many times the base is to multiplied by itself to indicate the value of the term.

$6^3 = 6 \bullet 6 \bullet 6 = 216$

exponential notation- (n) denoting (indicating) a certain value in terms of a base number with an exponent: nine as 3^2; forty-nine as 7^2; and two hundred sixteen as 6^3

$9 = 3 \bullet 3 = 3^2$ $\quad 49 = 7 \bullet 7 = 7^2$ $\quad 216 = 6 \bullet 6 \bullet 6 = 6^3$

power - (noun) 6^3 means 6 raised to the third power; 6 to the third power; the third power of 6; "power" indicates how many times a number or symbol is to multiplied by itself. For example, the value of term 6^3 is determined by multiplying 6 by itself three times. The value of 6^3 which is equal $6 \bullet 6 \bullet 6$ which is equal to 216.

Power of 10 - (n) any standard number is a power of ten if that has only one numeral one and the remaining numerals are zeros. The table two pages forward, The Most Frequently Used Numbers that are Powers of Ten, names the most frequently used numbers that are powers of ten.

product - (n) the answer when multiplying

NOTE: 3 times 5 = 3 X 5 = 3• 5 = 3 (5) = (3) 5 = (3)(5) = 15 15 is a product 3 & 5 are factors of 15

coefficient – (n) the number 5 is the coefficient of X in the term 5X; a coefficient is a constant.

factor - any numbers (a, b, c, ...) multiplied by each other form a product, and in turn are they factors of their product.

If 2•4•5 is multiplied, their product is 40 **2x4x5=40 or 2•4•5=40**

and 2, 4, and 5 are factors of 40

If a•b•c is multiplied, their product is abc **a•b•c= abc**

and a, b, and c are factors of abc

NOTE: 2•3 = 2 times 3 = 2 x 3 = (2)(3) = 2(3) = 2*3 = 6 6 is a product 2 & 3 are factors of 6

NOTE: 3 times 5 = 3 X 5 = 3• 5 = 3 (5) = (3) 5 = (3)(5) = 15 15 is a product 3 & 5 are factors of 15

NAMING VALUES WRITTEN IN EXPONENTIAL NOTATION

6^5	six to the fifth power 6 x 6 x 6 x 6 x 6 or	6 • 6 • 6 • 6 • 6
15^4	fifteen to the fourth power	15 • 15 • 15 • 15
7^3	seven to the third power or seven cubed	7 • 7 • 7
20^2	twenty to the second power or twenty squared	20 • 20
10^1	ten to the first power	10

CONCEPT If a product such as 25 has a factor such as 5 that when multiplied by itself (5 • 5), is equal to the product, then the product may be expressed in exponential notation, 5^2

Standard Notation	As a Product	Exponential Notation
32	= 2 • 2 • 2 • 2 • 2	= 2^5
25	= 5 • 5	= 5^2
343	= 7 • 7 • 7	= 7^3
10,000	= 10 • 10 • 10 • 10	= 10^4

Said another way, 32 is equal to the base 2 is multiplied by itself 5 times:

2^5 = 2 x 2 x 2 x 2 x 2 = 32 or 2^5 = 2 • 2 • 2 • 2 • 2 = 32

The exponent 5 tells how many times 2 is used as a factor in the product 32;

2 into 32 = 16	therefore 2 • 16 = 32
2 into 16 = 8	therefore 2 • 2 • 8 = 32
2 into 8 = 4	therefore 2 • 2 • 2 • 4 = 32
2 into 4 = 2	therefore 2 • 2 • 2 • 2 • 2 = 32

numbers (noun) are mental constructs (i.e., our inventions, our symbols) to describe
(1) how many objects are in a group. (i.e., the size of a group),
(2) the size of an individual object (how big or how small something is), and
(3) an item's position (rank or standing) among a group of items.

Decimal place , place value - a position that refers to a given power of ten in a number. The 4 in the number 4,000 is in the one thousand place and
equal to $4 \times 1000 = 4 \times 10 \times 10 \times 10 = 4 \bullet 10^3$

Decimal system - is the number system that we ordinarily use; it is based on powers of 10 and using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. A numeral in the decimal system is written as a row of digits, with each position in the row corresponding to a specific power of 10. In the table below, the most used powers of ten are given. Every listed number is a multiple of 10, and each number 10 times larger than the number listed on the line below it. Again, any standard number is a power of ten if that has only one numeral one and the remaining numerals are zeros.

The Most Frequently Used Numbers that are Powers of Ten Each power of ten is given in the following order: (a) in exponential notation, (b) as (factors of) a product, (b') as a common fraction, if applicable, (c) in standard notation, and d) in word description.

(a)	(b)	(b')	(c)	(d)
10^6	10 x 10 x 10 x 10 x10 x 10		1,000,000	one million
10^5	10 • 10 • 10 • 10 •10		100,000	one hundred thousand
10^4	10 • 10 • 10 •10		10,000	ten thousand
10^3	10 • 10 •10		1,000	one thousand
10^2	10 •10		100	one hundred
10^1	10		10	ten
10^0	10/10		1	one
•	decimal point	•	•	•
10^{-1}	1/10	1/10	.1	one tenth
10^{-2}	1/10 x 10	1/100	.01	one hundredth
10^{-3}	1/10 •10 • 10	1/1,000	.001	one thousandth
10^{-4}	1/10 • 10 •10 • 10	1/10,000	.0001	one ten thousandth
10^{-5}	1/10 • 10 • 10 •10 • 10	1/100,000	.00001	one one hundred thousandth
10^{-6}	1/10 • 10 • 10 • 10 •10 • 10	1/1,000,000	.000001	one one millionth

Decimal point - a dot used to mark the point at which place values change from positive to negative powers of 10 in the decimal number system; the negative powers of ten are fractions.

cardinal numbers describe the size (magnitude) of an object (measure how big it is) and cardinal numbers also count (enumerate how many) objects are in a group.

ordinal numbers refer to the relative position of an item in an ordered (or classified) group of items, such as first, second, third, etc.

finite numbers are numbers that can be counted; they have limits; they do not go on forever.

natural numbers are finite cardinal and ordinal numbers, represented by the numerals 1, 2, 3, . . .

integers include the natural numbers, their negatives and zero.

real numbers are in a one-to-one correspondence with all the points on a straight line.

imaginary numbers, rational numbers, and irrational numbers are described in another section.

Our Number System & Powers Of Ten (Number Sense)

Decimal - a number expressed in scales of 10, i.e., in powers of 10.

Decimal point - a dot written either on or slightly above the line; used to mark the point at which place values change from positive to negative powers of 10 in the decimal number system. Example: 63, 753, 284.823 is read: 63 million, 7 hundred fifty-three thousand, 2 hundred eighty-four and 8 hundred twenty-three thousandths

The extended notation (three more positions and values are added after the decimal,) of 63, 753, 284.823576 is given below:

60 million	= 6 • ten million		60,000,000
3 million	= 3 • million		3,000,000
7 hundred thousand	= 7 • hundred thousand		700,000
50 thousand	= 5 • ten thousand		50,000
3 thousand	= 3 • thousand		3,000
2 hundred	= 2 • hundred		200
80	= 8 • ten		80
4	= 4 • one		4
8 tenths	= 8 • one tenth	= 8 ÷ 10 = 8/10	.8
2 hundredths	= 2 • one hundredths	= 2 ÷ 100 = 2/100	.02
3 thousandths	= 3 • one thousandths	= 3/1,000	.003
5 ten thousandths	= 5 • one ten thousandths	= 5/10,000	.0005
7 hundred thousandths	= 7 • one one hundred thousandths		.00007
6 millionths	= 6 • one one millionths	= 6/1,000,000	+ .000006
			63,753,284.823576

Decimal fraction, **decimal number** - a number followed by decimal point followed by one or more digits; the digits right of the decimal represent a fraction where the denominator is a power of 10 and only the numerator is given in decimal notation.

Common fraction		Decimal fraction
1/100	=	.01
9/1000	=	.009

BASIC CONCEPTS AND DEFINITIONS

Firstly, to use the time-saving skill of multiplying or dividing by powers of ten, you must be able to recognize a number that is a power of ten. Numbers such as 10, 100, and 1000 are powers of ten because they are equal to 10^1, 10^2, and 10^3 respectively. One page back there is table listing the most used numbers that are powers of ten; if you are not yet sure about this concept review the table. Any standard number is a power of ten if that has only one numeral one and the remaining numerals are zeros.

Secondly, in any multiplication and division problems, the parts of the setup are as follow:

3	multiplicand	
X 4	X multiplier	multiplicand • multiplier = product
12	product	

$$\begin{array}{r}3\\3\overline{)12}\end{array} \qquad \begin{array}{r}quotient\\divisor\overline{)dividend}\end{array} \qquad \frac{dividend}{divisor} = quotient$$

Since a • b = b • a (example: 3 • 2 = 2 • 3) then either "a" or "b" may be named the multiplier. To be consistent and to make the following instructions as easy as possible, the number that is a POWER OF TEN is always named the MULTIPLIER.

TO MULTIPLY BY A POWER OF TEN,

first find the (given or implied) position of decimal point in the multiplicand, then move the decimal point to the right as many places from its original position as there are zeros in the multiplier.

Example A 2.74 • 10 = Since there is one zero in 10, move the decimal point in 2.74 one place right to yield the product 27.4

Then 2.74 • 10 = 27.4

Example B $ 57.84 • 100 = Since there is two zeros in 100, move the decimal point in 57.84 two places right to yield the product 5784.

Then $ 57.84 • 100 = $ 5784.00

Supply zeros as necessary to move the decimal point as many places as there are zeros in the multiplier (power of ten.)

Example C 657.4 • 1000 = Since there is three zeros in 1000, move the decimal point in 657.4 three places right, supplying zeros as necessary, to yield the product 657400

Then 657.4 • 1000 = 657400.

If the multiplicand has no given decimal, then it is implied to be just right of the numeral on the one's position. When any number has no decimal point, it is implied to be just right of the numeral in one's position; in the number 345, the "5" is in the one's position, therefore, the number with decimal point should be 345.

345• 10 = 345. • 10 86 • 100 = 86. • 100 5 • 10 = 5. • 10

Example D 57 • 100 = Since there is no given decimal, insert it just right of the 7, and since there are two zeros in 100, move the decimal point in 57 two places right, supplying zeros as necessary, to yield the product 5700.

Then 57 • 100 = $ 5700.

When multiplying a whole number by a power of ten as in the example above, you are effectively annex as many zeros to the right of the given multiplicand as there are zeros in the given multiplier. This is a special case; you should take advantage of it, but don't forget the concepts given above that work in every case.

Alternative Example D 57 • 100 = Since there are two zeros in 100, annex two zeros right of 57, supplying zeros as necessary, to yield the product 5700.

Then 57 • 100 = $ 5700.

Since we are dividing by the powers of ten, the number that is a POWER OF TEN is always named the DIVISOR.

TO DIVIDE BY A POWER OF TEN,

first find the (given or implied) position of decimal point in the dividend, then move the decimal point to the left as many places from its original position as there are zeros in the divisor.

Example A 27.4/10 = Since there is one zero in 10, move the decimal point in 2.74 one place left to yield the product 2.74

Then 27.4/10 = 2.74

Example B $ 5784./100 = Since there is two zeros in 100, move the decimal point in 57.84 two places left to yield the product 57.84

Then $ 5784.00/100 = $ 57.84

Supply zeros as necessary to move the decimal point as many places as there are zeros in the multiplier (power of ten.)

Example C 65.74/1000 = Since there is three zeros in 1000, move the decimal point in dividend 65.74 three places left, supplying zeros as necessary, to yield the product .06574

Then 65.74/1000 = .06574

If the dividend has no given decimal, then it is implied to be just right of the numeral on the one's position. When any number has no decimal point, it is implied to be just right of the numeral in one's position; in the number 345, the "5" is in the one's position, therefore, the number with decimal point should be 345.

Insert implied decimal point before dividing.

345/10 = 345./10 86/100 = 86./100 5/10 = 5./10

Example D 57/100 = Since there is no given decimal, insert it just right of the 7, and since there are two zeros in 100, move the decimal point in 57 two places left, supplying zeros as necessary, to yield the product .57

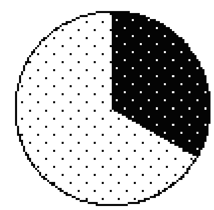
Then 57/100 = .57

A goal of this guide is to present a fractions review in the smallest space (fewest pages.)

Note: To do fractions quickly and accurately, you must know the multiplication facts.

The Meaning of a Fraction
Part of the confusion regarding fractions is that fractions have several meanings and uses.

First Meaning - A "fraction" may mean an amount less than all of something (one thing or a group of things.) The values (numbers) ½, ¾, and 7/8 are fractions. They could mean ½, ¾, and 7/8 of one pizza or ½, ¾, and 7/8 of the students in class today. In other words, a fraction may mean a part of one whole thing or a part of a group of things.



The pie in the figure was divided into three equal parts; each part was 1/3 of the whole pie: one divided by 3.

The shaded (dark) part represents 1/3 (one third) pie that is still in the pan, and the unshaded part represents 2/3 (two thirds or two 1/3 pieces) which are missing.

Note: 1/3 and 2/3 can be written as $\frac{1}{3}$ and $\frac{2}{3}$.

Another example is that the day is divided into 24 equal hours. One hour is equal to 1/24 of a day. We sleep about 8 hours at night or about 8/24 day or eight twenty-fourths day.

Second Meaning - A "fraction" may mean division.

The quotient for	also looks like this	and may be named (or called):
one divided by three	1/3	one third
two divided by three	2/3	two thirds
five divided by seven	5/7	five sevenths
one divided by two	1/2	one half
two divided by two	2/2	two halves
three divided by eight	3/8	three eighths
seven divided by ten	7/10	seven tenths
one divided by four	1/4	one fourth; one quarter
6 divided by 100	6/100	six hundredths

Other than halves and thirds, the denominator is denoted by a "th" on the end. If you do not speak, write and think the "th", you will have a great deal of trouble with fractions.

Third Meaning - A "fraction" may mean ratio.
Concept: If two quantities are compared by division, then the quotient (answer obtained) is called a ratio of the two quantities.
Concept: If a fraction means the ratio of two quantities, then the quantities must have the same unit. Say a worker has 5 days off every 2 weeks, but must work some weekends. The ratio of time (days) off would be 5/14 or 5:14: the units of the 5 and 14 must be the same.

It should be understood that ratios are expressed in several ways. For example 1 cup of bleach to 24 cups water in a mixture may be written as

1/24 bleach	the most frequent way to express a ratio is as a fraction with a division bar
1:24 bleach	another way to express a ratio is to use a colon rather than a division bar
.0416 bleach	the decimal number which is the quotient (answer) when dividing the common fraction's numerator by the denominator may be used to express a ratio
4.16% bleach	is a ratio expressed as a percent; 4.16% of mixture is water

Percent and ratios are presented in Section 5.

Related concepts and definitions:

Numerator – (n) is the part of a common fraction above the line; the numerator states (tells) the "number" of parts present in a fraction; the "3" is the numerator in the fraction 3/4. <u>The numerator names the number of pieces.</u>

Denominator – (n) is the part of a common fraction below the line; the denominator states (tells) how many equal parts the one (whole) is divided or broken into; 4 is the denominator in the fraction 3/4. The denominator names the size of the pieces: how the whole was divided.

Whole number – (n) is a number that has no fraction. Examples of whole numbers are 1, 2, 3, 35, 106, 147, and so on.

Mixed number – (n) is a number that is the sum a whole number and a fraction. Examples of mixed numbers are 6 1/2, 9 3/4, 3 ½, 2 ¾, and so on. Note: 1/2 and 3/4 may be written as ½ and ¾

<u>Improper fraction</u> – (n) is a fraction where the numerator is larger than the denominator. Improper fractions are like 5/2, 7/3, 8/4,6/2, 9/5 and so on. Numbers greater than one (1 whole something) can be written as a fraction, but usually it is improper (wrong) to leave answers that way. If you had 5 half dollars, you would have 2 1/2 dollars. 5/2 dollars = 2 1/2 dollars. You probably would not tell your friend that you had 5/2 dollars.

<u>Common fraction</u> – (n) is a fraction where the numerator is smaller than the denominator. Common fractions are also called proper fractions.

Least – (adjective) is a general-use word that means "smallest".

Common – (adjective) is a general-use word that frequently means "same".

Multiple – (n) is a product, the answer when one number is multiplied by another number. 2, 4, 6, 8, 10, 12, 14, and so on are multiples of 2.

<u>Concept of Adding and Subtracting Fractions with Unlike Denominators</u>

Fractions with different denominators, or unlike denominators, represent pieces of different sizes. In order to add or subtract fractions with unlike denominators, you need to change them to equivalent fractions with the same denominator. You can find equivalent fractions by either multiplying (or sometimes dividing) the numerator and the denominator of a fraction by the same nonzero number as shown on the next page.) The least common denominator, LCD, of two fractions is the least common multiple, LCM, of the two denominators.

LCM, <u>Least Common Multiple</u> – (n) is the smallest multiple that is common (same) for the given numbers. Example: the least common multiple of the given numbers 2 and 5 is 10. Multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, …; multiples of 5 are 5, 10, 15, 20, 25, 30, …

Note that 10, 20, 30, 40, 50, … are common multiples of 2 and 5, but 10 is the least common multiple.

LCD, <u>Least Common Denominator</u> – is the smallest denominator that two fractions with different denominators that both can be converted. When fractions are converted to the smallest equivalent fractions with the same denominator, then they have the LCD.

LCD, <u>Least Common Denominator</u> – is the smallest denominator that two fractions with different denominators that both can be converted. When fractions are converted to the smallest equivalent fractions with the same denominator, then they have the LCD.

Example: When $\frac{5}{8}$ and $\frac{1}{6}$ are converted to $\frac{15}{24}$ and $\frac{4}{24}$ they have the LCD.

$$\frac{5}{8} \qquad \frac{5}{8} * \frac{3}{3} \qquad \frac{15}{24}$$

$$+\frac{1}{6} \qquad +\frac{1}{6} * \frac{4}{4} \qquad +\frac{4}{24}$$

<u>Factor</u> – (n) in multiplication, each number multiplied is a factor of the product.
Example: 2 and 3 are factors of the product 6. Then if any given number (or product) has one or several factors, then that number (or product) is divisible by that/those same factor(s).

<u>Factoring</u> or to <u>factor</u> – (v) is part of the process to reduce (convert) a fraction to the equivalent fraction with the smallest numerator and denominator.

<u>Fractional numerals</u> = <u>Fractional form</u> = in the form of a fraction.

<u>Naming fractions</u>

1/3	one third	one divided by three
2/3	two thirds	two divided by three
3/3	three thirds	three divided by three; one
1/7	one seventh	one divided seven
5/7	five sevenths	five divided by seven
1/2	one half	one divided by two
2/2	two halves	two divided by two; one
3/8	three eighths	three divided by eight
3/10	three tenths	three divided by ten
7/10	seven tenths	seven divided by ten
1/4	one fourth; one quarter	one divided by four
2/4	two fourths; two quarters	two divided by four
57/100	fifty-seven hundredths	57 divided by 100
6/100	six hundredths	6 divided by 100

Other than halves and thirds, the denominator is denoted by a "th" on the end. If you do not speak, write and think the "th", you will have a great deal of trouble with fractions.

When naming mixed numbers "and" separates the whole number(s) from the fraction; the word "and" represents the decimal point.

6 1/2	six and one half
25 5/8	twenty-five and five eighths
4 2/9	four and two ninths
135 6/10	one hundred thirty-five and six tenths

MULTIPLICATION OF FRACTIONS

The purpose of this guide is to present a complete review in the smallest space/fewest pages.

NOTE: "Of" means times; of = • ; of = *

Problem: What is one half of one half? $\frac{1}{2} * \frac{1}{2} = ?$

Symbols	Meanings, descriptions and some examples
x, *, •, ( )	**multiply, multiplied by, multiplication:** **2x3 = 2*3 = 2•3= (2)3 = 2(3) = (2)(3) = 6**
÷ , /	**divide, divided by, division: 6 ÷ 3 = 6/3 = 2**

If a step does not apply (NA = not applicable), then go on to the next step.

Step A. If present, change mixed numbers and whole to fractions. If not present go to next step.

Step B. Multiply the numerator times the numerator and the denominator times the denominator. Step "B" is the actual multiplication operation.

Suggestion: First review the (written) steps to multiply fractions. Then pick a an example problem and perform steps A, B, C-1, and C-2 as applicable and reread each step as you go.

Example Problem 1	Step A	Step B	Step C-1	Step C-2
$\frac{1}{3} \bullet 18 =$	$\frac{1}{3} \bullet \frac{18}{1} =$	$\frac{1}{3} \bullet \frac{18}{1} = \frac{18}{3} =$	6	**NA**
Example Problem 2	**Step A**	**Step B**	**Step C-1**	**Step C-2**
$2\frac{1}{2} \bullet 3 =$	$\frac{5}{2} \bullet \frac{3}{1} = =$	$\frac{5}{2} \bullet \frac{3}{1} = \frac{15}{2} = =$	$7\frac{1}{2}$	**NA**
Example Problem 3	**Step A**	**Step B**	**Step C-1**	**Step C-2**
$\frac{3}{7} \bullet 1\frac{2}{3} =$	$\frac{3}{7} \bullet \frac{6}{3} =$	$\frac{3}{7} \bullet \frac{6}{3} = \frac{3 \bullet 6}{3 \bullet 7} =$	**NA**	$\frac{6}{7}$

Step C. Change answers to proper form.

Step C-1. Do not leave answer where the numerator is larger than the denominator. Convert (change) improper fractions to whole or mixed numbers.

Step C-2. Do not leave answer in fractional form where numerator and the denominator have a common factor. Reduce fractions to lowest terms (numbers.)

More about Step C-2. This procedure is sometimes called "reducing" fractions, sometimes called "simplifying" fractions, and sometimes "put in lowest terms." You would not tell a friend you had two quarter dollars, but rather you say you had one half dollar.
NOTE: one fourth = one quarter; two fourths = two quarters = one half

Two quarters written as a fraction is $\frac{2}{4}$ **then** $\frac{2}{4} = \frac{2*1}{2*2} = \frac{1}{2}$

DIVISION OF FRACTIONS

Step A. If present, change mixed numbers and whole numbers to fractions.

Step B. Rewrite the division problem as a multiplication problem.

Step B-1. Write the first fraction (1st reading from left to right)

Step B-2. Change the division sign to a multiplication sign.

Step B-3. Invert (turn upside down) the second fraction (2nd reading from left to right.)

More about Step B. Dividing by 4 is the same as multiplying by 1/4

Example Problem 1	Step A	Step B	Step C-1	Step C-2
$2\frac{1}{2} \div 4 =$	$\frac{5}{2} \div \frac{4}{1} =$	$\frac{5}{2} * \frac{1}{4} =$	$\frac{5}{8}$	NA

Step C. Finish the problem as though it was a multiplication problem. Refer to addition of fractions on the previous page.

Step C-1. Multiply (Refer to B on previous page.)

Step C-2. Reduce to proper form. (Refer to C-1 and C-2 on previous page.)

<u>Alternative Set up</u>

Alternate Step A. If present, change mixed numbers and whole numbers to fractions.

Example: To divide 2 1/2 by 4; convert 2 1/2 to 5/2 and convert 4 to 4/1

Alternate Step B. Write problem as a "double decker."

Alternative Example Problem 1	Alternative Step A	Alternative Step B	Alternative Step C	Alternative Step D
$2\frac{1}{2} \div 4 =$	$\frac{5}{2} \div \frac{4}{1} =$	$\dfrac{\frac{5}{2}}{\frac{4}{1}} =$	$\frac{5*1}{2*4} =$	$\frac{5}{8}$

Alternate step C. Multiplication step

Alternate step C-1. Multiply the outside numbers (absolute top and absolute bottom number) and then put a division symbol (a line) under it.

Alternate step C-2. Multiply the inside numbers, and place product under the division symbol.

Alternate step D. Finish the problem as though it was a multiplication problem. Refer to the addition of fractions on the previous page.

Alternate step D-1. Multiply (Refer to Multiplication Step B on the previous page.)

Alternate step D-2. Reduce to proper form.

(Refer to Multiplication Step C-1 and C-2 on the previous page.)

ADDITION & SUBTRACTION OF FRACTIONS

The easiest way to set up and work an addition or subtraction problem is in the vertical form; therefore, if a problem is given in the horizontal form, rewrite it in the vertical form.

Step A. If given in Horizontal form (written from left to right as the example below), **then rewrite it in Vertical form (written from top to bottom) as the example shown.→)**

5/9 + 2/9 = or $\frac{5}{9} + \frac{2}{9}$ **=**

$$\begin{array}{r} \frac{5}{9} \\ + \frac{2}{9} \\ \hline \end{array}$$

Step B. When denominators are different, convert (change) each fraction to an equivalent fraction with the Least Common Denominator (LCD) as the denominator. Fractions with different denominators, or unlike denominators, represent pieces of different sizes. For more, see "<u>Concept of Adding and Subtracting Fractions with Unlike Denominators</u>" on page 1.

$$\begin{array}{r} \frac{5}{8} \\ + \frac{1}{6} \\ \hline \end{array}$$

In the problem here, both 8 and 6 are denominators; and since 8, 16, 24, 32, 40, 48, 56, etc., are multiples of 8; and 6, 12, 18, 24, 30, 36, 42, etc., are multiples of 6; then 24 is the least common multiple of 8 and 6.

NOTE: the Least Common Denominator = least common multiple.

NOTE: Common denominators other than the least common denominator can be used, but using the LCD makes computing easier. For example 48 is another common multiple of 8 and 6.

To repeat in Step B, convert each fraction to an equivalent fraction with the Least Common Denominator (LCD) as the denominator; LCM = LCD

Step C-Add **To add fractions, keep the denominator and add numerator to numerator.**
Step C-Subtract **To subtract fractions, keep the denominator and subtract the numerator from numerator.**

Step A	Step B			C-ADD	C-SUBTRACT
$\frac{5}{8}$	$\frac{5}{8}*\frac{?}{?}$	$\frac{5}{8}*\frac{3}{3}$	$\frac{15}{24}$	$\frac{15}{24}$	$\frac{15}{24}$
$\pm\frac{1}{6}$	$\pm\frac{1}{6}*\frac{?}{?}$	$\pm\frac{1}{6}*\frac{4}{4}$	$\pm\frac{4}{24}$	$+\frac{4}{24}$	$-\frac{4}{24}$
				$\frac{19}{24}$	$\frac{11}{24}$

Step D. Add or subtract mixed numbers, **the first step (D-1) is to add or subtract the fractions (as shown above) then the second step (D-2) is add or subtract the whole numbers.**

Step E. Change answer to proper form. **Step E-1. Convert improper fractions to mixed or whole numbers.**
Step E-2. Reduce fractions to lowest terms.

Step D-1	Step D-2	Step E-1	Step E-2
$6\frac{7}{8}$	$6\frac{7}{8}$	$6\frac{7}{8}$	$6\frac{7}{8}$
$+2\frac{3}{8}$	$+2\frac{3}{8}$	$+2\frac{3}{8}$	$+2\frac{3}{8}$
$\frac{10}{8}$	$8\frac{10}{8}$	$9\frac{2}{8}$	$9\frac{1}{4}$

The primary use of the divisibility rules is reducing fractions to their lowest terms (proper form): for example, to convert $\frac{12}{36}$ to $\frac{1}{3}$. Therefore, for our purposes here, a number is divisible by another number only when it can be divided evenly (without a remainder) by that number.

15 ÷ 5 = 3 (no remainder); therefore, 15 is divisible by 5.

16 is not (evenly) divisible by 5 because 16 ÷ 5 = 3 plus a remainder of 1.

"Evenly divisible" means "divided without a remainder."

A number is "evenly divisible" by		Example	
2	If the number in the ones place is an even number.	24	since 24 ends with 4, an even number; then, 24 is (evenly) divisible by 2.
3	If the sum of its digits is divisible by 3.	168	since digits 1 + 6 + 8 = 15 and 15 ÷ 3 = 5, then 168 is divisible by 3: 168 ÷ 3 = 56
4	If the number made (formed) by the last two digits is divisible by 4	124	since 24, the last two digits of 124, is divisible by 4; then 124 is divisible by 4: 24÷ 4 = 6 and 124 ÷ 2 = 31
5	If the number in the ones place is a 0 or 5	130	since a "0" is in the ones place, then 130 is divisible by 5. 130 ÷ 5 = 26
6	If the number is divisible by both 2 and 3 (see rules for 2 and three above)	144	since ends in 4, an even number, and 1 + 4 + 4 = 9 and 9 ÷ 3 = 3, then 144 is divisible by 9: 144 ÷ 6 = 24
7	Eye don't no 1; do U?		
8	If the number made (formed) by the last three digits is divisible by 8	1568	since "568" is divisible by 8, then 1568 is divisible by 8 (without a remainder.) 568 ÷ 8 = 71 and 1568 ÷ 8 = 196
9	If the sum of the digits is divisible by 9	4374	since the sum 4 + 3 + 7 + 4 = 18 and 18 is divisible by 9, then 4374 is (evenly) divisible by 9. 4374 ÷ 9 = 486
10	If the digit in the ones place is 0.	780	since 780 has a "0" in the ones place, then 780 is divisible by 10. 780 ÷ 10 = 78

NOTE: 2 times 3 = 2 • 3 = 2 x 3 = (2)(3) = 2(3) = 2*3 = 6

Before you can understand decimal fractions, you must first understand pages Section 2, Our Number System & Powers Of Ten; review those pages if you have any doubt.

Decimal number system - the number system that we ordinarily use; it is a number system based upon the number 10; in theory, each unit is 10 times the next smaller one. In the list below, every given number is a multiple of 10, and each number 10 times the number listed on the line below it.

Decimal point – (n) a dot written either on or slightly above the line; used to mark the point at which place values change from positive to negative powers of 10 in the decimal number system.

10^3	= 10 • 10• 10	=	1,000	=	1000.	one thousand
10^2	= 10 • 10	=	100	=	100.	one hundred
10^1	= 10	=	10	=	10.	ten
10^0	= 10/10	=	1	=	1.	one
10^{-1}	= 1/10	=	1/10	=	.1	one tenth
10^{-2}	= 1/10 • 10	=	1/100	=	. 01	one hundredth
10^{-3}	= 1/10 • 10 • 10	=	1/1,000	=	. 001	one thousandth

decimal place or place value – (n) a position that refers to a given power of ten in a number. The 4 in the number 4,000 is in the one thousand place and is equal to 4 • 1000 = 4 • 10 • 10 • 10 = 4 • 103

decimal - a number expressed in scales of 10, i.e., in powers of 10.

decimal point - a dot written either on or slightly above the line; used to mark the point at which place values change from positive to negative powers of 10 in the decimal number system, in other words, from whole numbers to fractions.
Example: 63, 753, 284.823 is read: 63 million, 7 hundred fifty-three thousand, 2 hundred eighty-four and 8 hundred twenty-three thousandths

numerator - the part of a common fraction above the line; the numerator states (tells) the "number" of parts present in a fraction; 3 is the numerator in the fraction 3/4.

denominator - the part of a common fraction below the line; the denominator states (tells) how many equal parts the one (whole) was divided or broken into; 4 is the denominator in the fraction 3/4.

whole number - a number that has no fraction such as 1, 2, 3, 35, 106, 147, et al.

mixed number - a number that has both a whole number and a fraction such as 6 1/2, 9 3/4, 1 1/2 et al.

decimal fraction, decimal number - a number that has a decimal point; the numerals left of the decimal point are whole units, and the numerals right of the decimal represent a fraction where the denominator is a power of 10 and only the numerator is given in decimal notation

common fraction		decimal fraction	word description
3/10	=	.3	three tenths
6/100	=	.06	six hundredths
27/100	=	.27	twenty seven hundredths
9/1000	=	.009	nine thousandths
635/1000	=	.635	six hundred thirty five thousandths
mixed numbers		decimal numbers	word description
1 3/10	=	1.3	one and three tenths
7 15/100	=	7.15	seven and fifteen hundredths

ADDITION of decimal numbers

Place value - the value given to the position in which a digit appears in a number. In 368, 3 is in the hundreds place, 6 is in the tens place, and 8 is in the ones place.
Components of any addition problem are as given here:

```
  addend
+ addend
  sum
```

1. Write each addend so that the decimal points are directly under each other and digits for each place value are directly under each other. Zeros may be annexed to the numerals so that the addends may have the same number of decimal places.

Example : find the sum of .523 + 3 + 4.16 + 2.2 =

set up/arrange as	or set up/arrange as
0.523	0.523
3.	3.000
4.16	4.160
+ 2.2	+ 2.200
9.883	9.883

2. Add as in the addition of whole numbers.

3. Place the decimal point in the sum directly under the decimal points in the addends.

4. When a decimal answer ends in one or more zeros after the of the decimal point, the zeros may be drooped (unless it is necessary to show the exactness of measurements.)

```
  .388
+ .122
  .600 = .6
```

SUBTRACTION OF DECIMAL NUMBERS

Components of any subtraction problem are given here:

```
  minuend
- subtrahend
  difference
```

1. Write the subtrahend under the minuend so that the decimal points are directly under each other and digits for each place value are directly under each other. Zeros may be annexed to the numerals so that the minuend and the subtrahend have the same number of decimal places.

```
  5.61        5.61
-  .9       -  .90
  4.71        4.71
```

2. Subtract as in the subtraction of whole numbers

3. Place the decimal point in the difference (answer) directly under the decimal points in the subtrahend and minuend.

4. When the answer ends in one or more zeros to the right of the decimal point, the zeros may be dropped (unless it is necessary to show the exactness of measurements.)

```
  9.356
–  .856
  8.500 = 8.5
```

MULTIPLICATION of decimal numbers

1. Write the numbers and multiply exactly like the multiplication of whole numbers disregarding decimal placement. Place the numbers in straight, neat lines; the decimal points do not have to be arranged in any way.

2. After multiplying, find the sum of digits right of the decimal point in both the multiplicand (top number) and multiplier (bottom number), and counting from right to left, count off that total number of digits in the product.

```
  4.6          (1) (1 digit)
X .0 4       + (2) (2 digits)
 .184          (3) (3 digits right of the decimal)
```

3. Zeros may need to be supplied if there are not enough numerals (place values) in the product.

46	a whole number	(0) 46 is	.46	forty six hundredths	(2)
X 4	a whole number	+ (0) 4 is	X .04	four hundredths	+ (2)
184	a whole number	(0) §§	.0184	one hundred eighty-four ten thousandths	(4) §§

§§ Sum of the digits right of the decimal in both the multiplicand and multiplier.

DIVISION of decimal numbers

Components of any division problem are as follows:

dividend/divisor = quotient 18/6 = 3 eighteen divided by 6 is 3

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient} \qquad \frac{18}{6} = 3$$

$$\text{divisor}\overline{)\text{dividend}}^{\;\text{quotient}} \qquad 6\overline{)18}^{\;3}$$

1. IF the divisor is a whole number (i.e., no decimal point in the divisor), THEN:

a. Divide as in division of whole numbers, and

b. Place the decimal point in the quotient directly above the decimal point in the dividend.

9.12 / 8 = .015 / 5 =

$$8\overline{)9.12}^{\;1.14} \qquad 5\overline{).015}^{\;.003}$$

2. IF the divisor has a decimal point between two digits,

THEN	Example 1	Example 2
a. Make the divisor a whole number by moving its decimal point to the right of the last digit, indicating it's new position by a caret "↓."	$.4\downarrow\overline{)35.6}$	$\$1.50\downarrow\overline{)6.00}$
b. Move the decimal point in the dividend to the right an equal number of places as you moved the decimal point in the divisor.	$.4\downarrow\overline{)35.6\downarrow}$ moved 1 digit	$\$1.50\downarrow\overline{)6.00\downarrow}$ moved 2 digits
c. Divide as in the division of whole numbers and place the decimal point in the quotient directly above the caret "↓." in the dividend.	8 9. $.4\downarrow\overline{)35.6\downarrow}$	4. $\$1.50\downarrow\overline{)6.00\downarrow}$

3. To check your answer, multiply the quotient by the divisor and add the remainder, if any, to the product. Note the decimal point of the divisor is in its original position (not at the caret position) when multiplying.

	digits		digits
89	(0)	\$1.50	(2)
<u>.4</u>	<u>(1)</u>	<u>.4</u>	<u>(1)</u>
35.6	(1)	\$6.00	(3)

4. To find the quotient correct to the nearest required decimal place, find the quotient to one more than the required number of decimal places, then round off.

IF problem is to solve to nearest:	THEN work problem through the:
tenths	hundredths place value & rounds to tenths
hundredths	thousandths place value & round to hundredths
thousandths	ten thousandths place value & round to thousands
ten thousandths	hundred thousandths place value & round to thousandths

Example problem: Divide 28.5 by .87 and find the quotient to the nearest tenth.

Recommended Method

To calculate answer to the nearest tenth, solve through hundredths place value then round off to the nearest tenth

23. 75 ≈ 23. 8

Example $.87 \downarrow \overline{)28.50 \downarrow 00}$

See problem worked below

In this example, solve quotient through the hundredth position (place value,) then round off .75 to .8 because the number in the hundredths position ≥ 5 The true answer is nearer to 32.8 than 32.7

```
         23. 75 ≈ 23. 8
.87↓ )28.50↓00
      26 11
         24 0
         17 0
          6 60
          6 09
            510
            435
             75
```

Alternative Method

To calculate answer to the nearest tenth, solve through tenth place value then add 1 to the tenth place value if the remainder ≥ 1/2 of the divisor or subtract 1 from the tenth place value if the remainder ≤ 1/2 of the divisor.

23. 7 + .1 ≈ 23. 8

Example $.87 \downarrow \overline{)28.50 \downarrow 0}$

See problem worked below

In this example, solve quotient through tenth position (place value,) then add 1 to the tenth place value since 51 is greater than 1/2 of 87. The true answer is nearer to 32.8 than 32.7

```
         23. 75 ≈ 23. 8
.87↓ )28.50↓00
      26 11
         24 0
         17 0
          6 60
          6 09
             51
```

One more DIVISION example: 24.48 / 0.8 = ?

IF the divisor has a decimal point between two digits, THEN

Step a. Make the divisor a whole number by moving its decimal point to the right of the last digit, indicating it's new position by a caret "↓."

.8↓)24.48

In example, move the decimal point in .8 over one place value so that you will be dividing by the whole number 8; in the problem, the new decimal point location is indicated by a caret ↓

moved 1 digit in divisor

Step b. Move the decimal point in the dividend to the right an equal number of places as you moved the decimal point in the divisor

.8↓)24.4↓8

In example, move the decimal point in 24.48 one place value so it is now 244.8; in the problem, the new decimal point location is indicated by a caret ↓.

moved 1 digit in dividend

Step c. Divide as in the division of whole numbers and place the decimal point in the quotient directly above the caret "↓." in the dividend.

```
     3 0 . 6
.8↓)24.4↓8
```

NOTE: Moving the decimal point the same number of places in the divisor and dividend does not change the answer since this is really the same as multiplying the quotient (the answer) by 1 HOW CAN THAT BE? Let's consider the example on this page.

In the example, since 24.48/.8 • 10/10 = 244.4/8 and since 10/10 = 1, the value is unchanged. To say again:

$$\frac{24.48}{.8} = \frac{24.48}{.8} \bullet \frac{10}{10} = \frac{24.48 \bullet 10}{.8 \bullet 10} = \frac{244.8}{8} = 3\,0.6$$

NOTE: Don't forget the zero in this quotient; 8 is not contained in 4.

```
     3 0. 6
.8↓)24.4↓8
    24
      04 8
       4 8
         0
```

Topic 1: Percent

While you can work percent problems by simply following the instructions here, real understanding of these procedures requires knowledge of both common fractions and decimal fractions. Review those topics as needed.

CONCEPT: percent is a fraction with a denominator of 100.

Value	**50%**	**= 50 parts/100 parts**	**= .50**	**= 50/100**	**= 1/2**
Read as	**fifty percent**	**50 parts per 100 parts or 50 parts for every 100 parts**	**fifty hundreds**	**fifty hundreds**	**one half**

Since $\frac{1}{2} = \frac{(1)}{(2)} = \frac{(1)(50)}{(2)(50)} = \frac{50}{100} = .50 = 50\%$

then fifty percent of something is the same as fifty hundredths of something which is the same as one half of something, (and also the same as five tenths of something.)

50% of 48 = 24	**fifty percent of 48 is equal to 24**
.50 • 48 = 24	**fifty hundredths times 48 is equal to 24**
50/100 • 48 = 24	**fifty hundredths times 48 is equal to 24**
1/2 of 48 = 24	**one half of 48 is equal to 24**
also .5 • 48 = 24	**five tenths times 48 is equal to 24**
also 5/10 • 48 = 24	**five tenths times 48 is equal to 24**

Common equivalents you should memorize.

percent	decimal form	fractional form
12.5%	.125	1/8
20%	.20	1/5
25%	.25	1/4
33%	.33	1/3
50%	.50	1/2
67%	.67	2/3
75%	.75	3/4
100%	1.00	1
150%	1.50	1 1/2

CONCEPT **Since** $percent = \frac{parts}{100\,parts} = \frac{1\,part}{100\,parts} = \frac{1}{100} = .01$

therefore .01 can be substituted for % when working percent problems.

You can work percent problems by MAKING SIMPLE SUBSTITUTIONS.

word or symbol	Meaning	Example
%	"parts per 100 parts".	50% = 50 parts/100 parts
of	"times" or "multiplied by."	1/2 of 10 = (1/2)(10) = 5
per	"for each" or "for every"	a pint of milk per child
per	"divided by"	30 miles per hour or $\frac{30\text{ miles}}{\text{hour}}$ or 30 miles/hour or 30 mph

PROCEDURES TO CONVERT percents, fractions, and decimal numbers to other forms of equal value

Procedure #1 : to convert from percent to decimal form
(1) drop the percent sign, and
(2) move the decimal point 2 digits to the left
Example: 60% = .60

Procedure #2: to convert from percent to fractional form
(1) drop the percent sign, and
(2) supply a denominator of 100
Example: 60% = 60/100 = 3/5

Procedure #3: to convert from decimal to percent form
(1) move decimal point 2 digits to the right, and
(2) supply a percent sign
Example: $.32 = .32\downarrow\% = 32\%$

Procedure #4: to convert fractional form to percent form
(1) first convert fraction to decimal form by either
(a) dividing the numerator (top) by the denominator to two decimal places, OR
(b) multiplying the fraction times another fraction so that the denominator is 100, then rewrite as a decimal numeral
(2) then apply Procedure #3 to convert decimal to percent form

Example 4a $\frac{3}{25} = .12 = .12\downarrow\% = 12\%$ $\qquad 25\overline{)3.00}$ = .12

Example 4b $\frac{3}{25} = \frac{3*4}{25*4} = \frac{12}{100} = .12 = 12\%$

Topic 2: percent sentences

In each of these problem situations, the set up is the same basic sentence:
___ is ____ % of ___ ? (blank is what percent of blank?)

While working the problems,

for word or symbol	substitute (or insert)
%	.01
of	substitute multiplication symbol • or * or insert ( ) appropriately
is	=
per	/ or ÷

Example A

___ is ____ % of ___ ? (blank is what percent of blank?)

What is 5% of 80?	NOTES & PROCEDURES
A = 5% of 80?	A = answer; answer could be represented by any letter
A= (5)(.01)(80)	substitute .01 for % and recognize that "of" means times
A= 4	(5)(.01)(80) = 4
Then 4 is 5% of 80.	

Example B

___ is ____ % of ___ ? (blank is what percent of blank?)

4 is 5% of what number?	NOTES & PROCEDURES
4 is 5% of A?	A = answer; answer could be represented by any letter
4 = (5)(.01) A	substitute .01 for % and recognize that "of" means times
4 = .05A	(5)(.01) = .05
$\frac{4}{.05} = \frac{.05}{.05}A$	to isolate the answer "A", divide both sides of the equation by .05
	$05\downarrow\overline{)4.00\downarrow}$ quotient $80.$

Since A = 80,
Then 4 is 5% of 80

Example C

___ is ____ % of ___ ? (blank is what percent of blank?)

4 is what % of 80?	NOTES & PROCEDURES
4 is A % of 80?	A = answer; answer could be represented by any letter
4 = A (.01) 80	substitute .01 for % and recognize that "of" means times
4 = .80 A	to isolate the answer "A", divide both sides of the equation by .80
<u>A = 5</u> →	<u>This is only a partial solution; it is not the solution!</u>
4 is what % of 80	Original problem
Then 4 is 5% of 80	Substitute "5" for "what" <u>Solution is 5%</u>

Topic 3: ratios and proportions

CONCEPT: Definition of ratio - when two quantities are compared by division, the answer obtained is called a ratio of the two quantities

CONCEPT: It should be understood that ratios are expressed in several ways. For example 1 cup of bleach to 24 cups water in a mixture may be written as

1/24 bleach	**the most frequent way to express a ratio is as a fraction with a division bar**
1:24 bleach	**another way to express a ratio is to use a colon rather than a division bar**
.0416 bleach	**the decimal number which is the quotient (answer) when dividing the common fraction's numerator by the denominator may be used to express a ratio**
4.16% bleach	**is a ratio expressed as a percent; 4.16% of mixture is water**

As stated above, since $percent = \frac{parts}{100\,parts}$, **then percent is a ratio**

CONCEPT: Percent is the ratio of some number to one hundred.

Parts chlorine to parts water in mixture may be written as
5:1,000,000 chlorine to water or
5:1,000,000 or
.000005 or
.0005% (a ratio expressed as a percent)

Examples of other ratios or rates are:

$$\frac{60\,miles}{hour} = 60\frac{miles}{hour} = 60\,mph$$

$$\frac{60\,miles}{4\,hour} = \frac{15miles}{hour} = 15\frac{miles}{hour} = 15mph$$

PERCENT REVIEW: Remember that the symbol % indicates

(1) parts per hundred parts, for example,

(2) 70% represents a fraction where 70 is the numerator (top) and 100 is the denominator (bottom) of a fraction: $70\% = {}^{70}\!/_{100} = .70 = .7 = {}^{7}\!/_{10}$

70%	**of 400**	**= 280**
.70	**of 400**	**= 280**
70/100	**of 400**	**= 280**
.7	**of 400**	**= 280**
7/10	**of 400**	**= 280**

Word problems

PROBLEM: If you drove 400 miles yesterday and you increased the miles drove today by 20%, how many miles did you travel today?

D_Y = distance drove yesterday

D_T = distance drove today

I = increase in miles drove

$$D_T = D_Y + I$$

First compute the increase, I. What is 20% of 400 miles?

Increase = I = 20 • .01 • 400 miles

I = 80 miles

Then add the increase to miles drove yesterday

$$D_T = D_Y + I$$

$$D_T = 400\,miles + 80\,miles = 480\,miles$$

PROBLEM: A few months ago you could only do 15 sit-ups and now you are able to do 60 sit-ups. What is the percent of change (increase in this case) ?

change	= improvement	= difference = 60 - 15	= 45	
base	= benchmark	= beginning point		= 15
% change	= change/base	= 45/15	= 3	= 300%

PROBLEM: If you could do 50 sit-ups last month and you are now able to do 60 sit-ups this month. What is the percent of change (increase in this case) ?

change	= improvement	= difference = 60 - 50	= 10	
base	= benchmark	= beginning point	= 50	
% change	= change/base	= 10/50	= .20	= 20%

PROBLEM: On January 1 Diane was hired as a bookkeeper at the salary of $850.00 per month. On April 1 and on October 1 she received 10% raises. What was her gross earnings for the year?

During the year Diane received three different salaries, so the problem must be broken down into several problems.

S_J = Jan Salary = $850.00

S_A = Apr Salary = Jan Salary + 10% increase (NOTE: "increase" implies addition)
= $850.00 + (10% • $850.00)
= $850.00 + $85.00 = $935.00

S_O = Oct Salary
= Apr Salary + 10% increase
= $935.00 + (10% • $935.00)
= $935.00 + $93.50 = $1028.50

S_G = Gross salary for year
= (3 • $850.00) + (6 • $935.00) + (3 • $1028.50)
= $2550.00 + $5700.00 + $3085.50
= $ 11,355.50

Topic 4: proportions

In general use a proportion is a relationship between things or parts of things with respect to comparative magnitude, quantity, or degree: she has suitable height-weight proportions or the proper proportion between bleach and water in a cleaning mixture.

In mathematics a proportion is statement of equality between two ratios. The four quantities, a, b, c, d, are said to be in proportion if $\frac{a}{b}=\frac{c}{d}$

For example, if triangle ABC is similar to triangle XYZ and side a = 3, b = 4, c = 5, x = 9, and y = 12, then proportions can be used to calculate side z. Side 'a' is to side 'x' as side 'c' is to side 'z'.

Side '*a*' is to side '*x*' as side '*c*' is to side '*z*' means: $\frac{a}{x}=\frac{c}{z}$

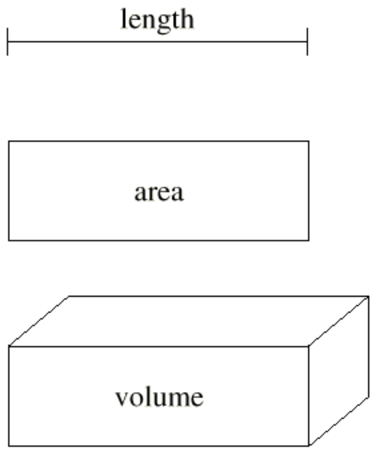
$$\frac{a}{x}=\frac{c}{z}$$

Side '*a*' in the small triangle and side '*x*' in the larger triangle are corresponding parts;
side '*c*' in the small triangle and side '*z*' in the larger triangle are corresponding parts. Both ratios are parts in small triangle to corresponding parts in the larger triangle.
You can switch them, but <u>both ratios must be the same</u>: small to large or large to small.

$$\frac{3}{9}=\frac{5}{z}$$

$$z=\frac{5(9)}{3}=15$$

Notice that both $\frac{3}{9}$ and $\frac{5}{15}$ can be reduced to $\frac{1}{3}$.



Length (n) is the distance between two points. Length is a one-dimensional measurement expressed in linear units. Some examples of linear units are:
inches, feet, yards, centimeters, and kilometers otherwise shown as in., ft, yd, cm, and km.

Area (n) is the number of square units in a bounded plane. Area is a two-dimensional measurement expressed in square units. Some examples of square units are:
square inches, square feet, square yards, square centimeters, and square kilometers otherwise shown as in^2, ft^2, yd^2, cm^2, and km^2.

Volume is the size of the region *inside* a solid. Volume is a three-dimensional measurement expressed as cubic units. Some examples of cubic units are:
cubic inches, cubic feet, cubic yards, cubic centimeters, and cubic kilometers otherwise shown as in^3, ft^3, yd^3, cm^3, and km^3.

standard unit - (n) any fixed quantity (numerical amount) considered as an accepted standard to measure something; examples of linear units include inch, foot, yard, mile, meter, centimeter, and kilometer; other units include degree, hour, minute, ampere, decibel, horsepower and many, many more.

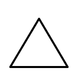
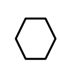
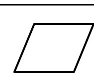
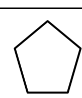
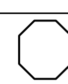
measurement - (n) the number of standard units between two points; a measurement, therefore, must include a number (amount) and the unit (used as the standard) such as 3 feet, 10 miles.

dimension - (n) the measurement in a straight line of the breadth, height, or thickness of something. If an entity has more than one dimension, then each measurement must be at a right angle (90-degree angle) to measurement(s) already taken.

bounded plane - (n) is a plane with boundaries; it is a two-dimensional and the sides are connected (closed); it has both breadth and height but no thickness, but no thickness (depth) because a plane is "flat."

plane figures (shapes) - (n) are closed (connected) sets of points in a plane that have an interior and an exterior (an inside and a outside.) They have breadth (width) and height, but no depth (thickness); therefore, they have two dimensions, 2-D, and surface areas that can be measured in square units.

A. circle - A plane curve everywhere equidistant from a given fixed point, the center.
B. polygon - A <u>closed</u> plane figure bounded by three or more line segments.

Numbe r of sides	Polygon	Example shape	Numbe r of sides	Polygon	Example shape
3	triangle tri = three		6	hexagon hex = six	
4	quadrilateral quad = four		7	heptagon hept = seven	
5	pentagon pent = five		8	Octagon Oct = eight	

♦♦ Statements that begin with a double asterisk are abstract concepts for advanced students.

geometry - (n) a branch of mathematics that deals with the measurement of the shape, size, and other properties of geometric figures.

figure - (n) in geometry, a figure is closed (connected) set of points that has an interior and an exterior (an inside and an outside.) Examples plane figures include triangle, square, and circle. ♦♦According to this definition, a line is not a figure, but sometimes a line is referred to as a figure.

figure - (n) in arithmetic, a figure is a symbol for a whole number such as "3", "4", and "23".)

standard - (n) anything recognized or accepted as correct and used as a basis of comparison

standard unit - (n) any fixed quantity (numerical amount) considered as an accepted standard to measure something; examples of linear units include inch, foot, yard, mile, meter, centimeter, and kilometer; other units include degree, hour, minute, ampere, decibel, horsepower and many, many more.

measure - (v) to define (find) the distance between two points.

measurement - (n) the number of standard units between two points; a measurement, therefore, must include a number (amount) and the unit (used as the standard) such as 3 feet, 10 miles.

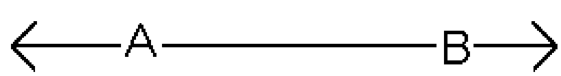
dimension - (n) the measurement in a straight line of the breadth, height, or thickness of something. ♦♦If an entity has more than one dimension, then each measurement must be at a right angle (90 degree angle) to measurements already taken.

congruent - (adjective) of or having the same size and shape.

point - (n) a location (having a specific position), but having no dimension, 0-D, (no breadth, height, or thickness.) We represent (draw a model of) a point with a dot. ♦♦We can see our representation (the dot), but since in reality a point is a location, it cannot be seen.

line - (n) contains more than one point on a straight path; extends in opposite directions indefinitely (forever.) ♦♦A line is a concept (a mental construction), and, therefore, cannot be seen. We CAN represent (draw a model of) a line and identify (name) it by two points on the representation (model.) A line containing points A and B can be named line AB or

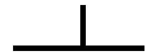
AB Model:



The arrows indicate that the line extends indefinitely.

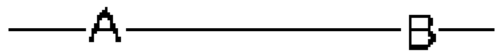
parallel lines - lines that do not intersect no matter how far extended, and, by definition, are equal distance apart.

Model: 

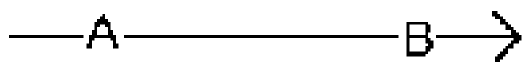
perpendicular lines - lines that intersect and form right angles. Model: 

linear - (adjective) meaning alone a line

segment - (or line segment) a part of a line consisting of two endpoints and all the points between them. Since a segment has endpoints, it has breadth, but no height or thickness; therefore a segment has one dimension, 1-D. A segment containing endpoints X and Y can be identified/named segment XY or

AB Model: 

ray - a part of a line consisting of one endpoint and an extension indefinitely (forever) in the opposite direction (from the endpoint.) A ray containing endpoint A can be identified/named ray AB or

AB Model: 

plane - is everywhere flat, it extends indefinitely in all directions "on" it, and its position (location) is determined by any one of the following four conditions:

(1) three points not on a line
(2) a line and a point not on that line
(3) two intersecting lines
(4) two parallel lines

A plane is a concept (mental construction), and, therefore, cannot be seen. We CAN represent (draw a model of) a plane and identify/name it plane P, plane T, and so on.

plane surface or bounded plane - a two-dimensional, connected (closed) portion of a plane; a plane with boundaries; it has breadth and height, but no thickness (depth) because a plane is "flat."

plane figure - (n) a two-dimensional geometric figure (having breadth and height, but no thickness/depth.) A closed figure with all points in the same plane such a triangle, rectangle, square, and a circle

space - (n) extends in all directions without bound (limits) and contains all points, lines, and planes.

♦♦Space is a concept and, therefore cannot be seen.

space figure - also called a solid, is a three-dimensional geometric figure or portion of space that has breadth, height, and thickness (depth.) Examples include cones, prisms, polyhedrons, and spheres.

distance - (n) is the measurement between two points in linear units; has one dimension. Since it is "one" dimension, it has a number and a linear unit. Examples of linear units include inches, feet, yards.

perimeter - (n) distance around a closed figure in a plane.

surface - (n) the outer face or exterior of something; the set of points that has breadth and height but no thickness.

area - (n) the measurement of the amount of surface of something in square units; has two dimensions: length and width. Since it has two dimensions, it has a number it has a number and a square unit, and it "is" the number of square units (or units squared.) Square units are obtained by multiplying (linear unit) times (linear unit).

Examples include square inches (sq in), square feet (sq ft), and square yards (sq yd.)

$(ft)(in) = (in)^2 = (sq\ in)$

square - (n) is a plane figure having four equal sides and four 90 degree angles.

square - (adjective) describes a unit of measurement of area and/or having two dimensions.

squared - (v) to raise the second power by multiplying (a number or quantity) times itself.

volume - (n) the measurement of the amount of space in something in cubic units; has three dimensions: length, width and depth. **Since it has three dimensions, it has a number and a cubic unit, and it "is" the number of cubic units (units cubed.) Cubic units are obtained by multiplying (linear unit) times (linear unit) times (linear unit).

Examples include cubic inches (cu in), cubic feet (cu ft), and cubic yards (cu yd.)

$(in)\ (in)\ (in) = (in)^3 = (cu\ in)$

cube - (n) is a space figure with six congruent square faces.

cubic - (adjective) describes a unit of measurement of space and/or having three dimensions.

cubed - (v) to raise to the third power by multiplying by itself by itself by itself or by multiplying it by its square.

Example 1: (4) raised to the third power = $(4)\ (4)\ (4) = (4)^3 = 64$

Example 2: (4) raised to the third power = $(4)\ (4)^2 = (4)^3 = 64$

mass - is not a linear measurement as those above are; mass is a quantity of matter (material) that an object contains; the weight of an object can change with the distance from the center of the earth even though its quantity of the material is unchanged.

1 cc (cubic centimeter) of water weights 1 gram; in other words, there are connections among metrics units which don't exist among Imperial (English) units

Concept	Measurement is a:	Dimensions	Standard	Derivation
point	N/A, since it is a location	0	N/A	N/A
line	distance	1	linear unit	N/A
plane	(surface) area	2	square unit	= unit • unit
space	volume	3	cubic unit	= unit • unit • unit

N/A = Not Applicable, does not apply, not connected with the matter at hand.

Concept	Figures	Unit	Example Units
line	N/A	linear unit	inch (in), foot (ft), yard, mile, millimeter (mm), centimeter (cm), meter (m), kilometer (km)
plane	circle, square triangle, rectangle	square unit, or unit squared, or $unit^2$	sq in, sq ft, sq yard, sq mile, sq mm, sq cm, sq meter, sq km, cm • cm = (cm)(cm) = sq cm = cm^2 ft * ft = sq ft = ft^2
space	sphere, cube, prism, cylinder	cubic unit, or unit cubed, or $unit^3$	cubic in, cubic ft, cubic yd, cu mile, cu mm, cubic cm, cu meters, cu km, cm * cm * cm = cubic cm = cm^3 cm^3 = cc

You are expected to pick the appropriate formula from the provided Reference sheet on tests; see page example Junior High Reference Sheet on the next page.

PLANE FIGURE	PERIMETER	SURFACE (AREA)
Rectangle	$P = 2(S_1+S_2)$	A = base*depth = bd
Square	P = 4S	A = side*side = S^2
Triangle	$P = S_1+S_2+S_3$	A = 1/2 b*h =bh/2
Circle	# $P = 2\pi r = d\pi$	$A = \pi r^2$
	# Called a circumference	

SOLID FIGURE	VOLUME	SURFACE (AREA)
Rectangular Prism	V = base*depth*height V = bhd = Ah	$A_{Total} = 2A_1+2A_2+2A_3$
Triangular Prism	V = Ah where A = 1/2 b*d	$A_{Total} = 2A_{top}+A_{S1}+A_{S2}+A_{S3}$
Cube	$V = S^3$	$A = 6\ S^2$
Sphere	$V = 4/3\ \pi r^3 = 1/6\ \pi d^3$	$A = 4\ \pi r^2 = \pi d^2$
Cylinder, right circular	$V = \pi r^2 h$	$A = 2\ \pi rh$
Cone, right circular	$V = 1/3\ \pi r^2 h$	

MEASUREMENT & GEOMETRY

mass @	Imperial Units	Metric
	ounces (oz) pounds (lbs) (#) tons	milligram (mg) gram kilogram (kg)

@ Mass is not a linear measurement

mass is the quantity of matter (material) an object contains and therefore a constant; whereas the weight of an object is a variable because it changes with the distance from the center of the earth even when its quantity of the material is unchanged.

Junior High Formula Reference Sheet

(Figure)	Perimeter	Area
Parallelogram	$P=S_1+S_2+S_3+S_4$	$A=b\bullet h$
Rectangle	$P=S_1+S_2+S_3+S_4$ or $P=2l+2w$	$A=b\bullet h$
Square	$P=4S$	S^2
Triangle	$P=S_1+S_2+S_3$	$A=½\, b\bullet h$
Trapezoid	$P=S_1+S_2+S_3+S_4$	$A=½\,(b_1+b_2)h$
Circle	Circumference $C=2\pi r$	$A=\pi r^2$
		Volume
Rectangular prism		$V=lwh$
Rectangular pyramid		$V=1/3\ lwh$
Cylinder		$V=\pi r^2 h$
Cone		$V=1/3\ \pi r^2 h$
Sphere		$V-=4/3\ \pi r^2$

Key

b = base h = height	l = length w = width s = side	r = radius d = diameter Use 3.14 for π

STANDARD UNITS OF MEASUREMENT: American, metric, conversion factors

Review

standard unit - (n) any fixed quantity (numerical amount) considered as an accepted standard to measure something; examples of linear units include inch, foot, yard, mile, meter, centimeter, and kilometer; other units include degree, hour, minute, ampere, decibel, horsepower and many, many more.

measurement - (n) the number of standard units between two points; a measurement, therefore, must include a number (amount) and the unit (used as the standard) such as 3 feet, 10 miles.

dimension - (n) the measurement in a straight line of the breadth, height, or thickness of something. **If an entity has more than one dimension, then each measurement must be at a right angle (90 degree angle) to measurements already taken.

linear - (adjective) meaning alone a line

Linear Units (1-D, 1 dimension, or, 1 measurement)

Table of Metric Linear Units

10 millimeters (mm)	= 1 centimeter (cm)	
10 centimeters	= 1 decimeter (dm)	= 100 millimeters
10 decimeters	= 1 meter (m)	= 1,000 millimeters
10 meters	= 1 dekameter (dam)	
10 dekameters	= 1 hectometer (hm)	= 100 meters
10 hectometers	= 1 kilometer (km)	= 1,000 meters

Table of American (British) Linear Units

12 inches (in)	= 1 foot (ft)	
3 feet	= 1 yard (yd)	
5 1/2 yards	= 1 rod (rd), pole, or perch	= 16 1/2 feet
40 rods	= 1 furlong (fur)=220 yards	= 660 feet
8 furlongs	= 1 statute mile (mi) = 1,760 yards	= 5,280 feet
3 miles	= 1 league = 5,280 yards	= 15,840 feet
6076.11549 feet	= 1 International Nautical Mile	

Table of Conversion Factors: Metric to American (English) Linear Units

1 millimeter (1 mm)	= 0.04	inches (in)
1 centimeter (1 cm)	= 0.4	inches (in)
1 meter (1 m)	= 3.3	feet (ft)
1 meter(1 m)	= 1.1	yards (yd)

STANDARD UNITS OF MEASUREMENT: American, metric, conversion factors

Area Units (2-D, or dimensions, or, 2 measurements)

Table of Metric Units of Area

1 square centimeter, 1 centimeter squared (sq cm, cm^2)	= 1/10,000 square meter
1 square decimeter, 1 decimeter squared (sq dm, dm^2)	= 1/100 square meter
1 square meter, 1 meter squared (sq m; m^2	basic unit of area
1 are (a)	= 100 square meters
1 hectare (ha)	= 10,000 square meters
1 square kilometer, 1 kilometer squared (sq km, km^2)	=1,000,000 square meters

Table of American (British) Units of Area

1 square inch, 1 inch squared (sq in, in^2)	= 1/1,296 sq yard	= 1/144 sq foot
1 square foot, 1 foot squared (sq ft, ft^2)	= 1/9 sq yard	= 144 sq inches
1 square yard, 1 yard squared (sq yd, yd^2)	= 9 sq feet	
1 square rod, 1 rod squared (sq rd, rd^2)	= 30 1/4 sq yards	
1 acre	= 4,840 sq yards	= 160 sq rods
1 square mile, 1 mile squared (sq mi, mi^2)	= 3,097,600 sq yards	= 640 acres

Table of Conversion Factors for Units of Area

1 square inch, 1 inch squared (sq in, in^2)	= 6.45 square centimeters
1 square foot, 1 foot squared (sq ft, ft^2)	= 0.093 square meter
1 square yard, 1 yard squared (sq yd, yd^2)	= 0.84 square meter
1 acre	= 0.405 hectare
1 square mile, 1 mile squared (sq mi, mi^2)	= 2.59 square kilometers
1 square centimeter, 1 centimeter (sq cm, cm^2)	= 0.155 square inch
1 square meter, 1 meter squared (sq m, m^2)	= 1.2 square yards
1 hectare	= 2.47 acres
1 square kilometer, kilometer squared (sq km, km^2)	= 0.386 square mile

STANDARD UNITS OF MEASUREMENT: American, metric, conversion factors

Units of Volume (3-D, or 3 dimensions, or, 3 measurements) and Capacity, Liquid and Dry

Table of Metric Units of Volume and Capacity

1 cubic centimeter (cc)	= 1/1,000,000 cubic meter	
1 cubic decimeter (cu dm)	= 1/1,000 cubic meter	= 1,000 cubic decimeters
1 cubic meter (cu m)	= 1 stere (s)	
1 milliliter (ml)	= 1/1,0000 liter	= 1 cubic centimeter
1 centiliter (cl)	= 1/100 liter	= 10 milliliters
1 deciliter (dl)	= 1/10 liter	
1 liter	= 1 cubic decimeter	
1 dekaliter (dkl)	= 10 liters	
1 hectoliter (hl)	= 100 liters	= 1/10 cubic meter
1 kiloliter (kl)	= 1,000 liters	

American and British Units of Volume and Capacity

1 cubic inch (cu in.)	= 1/46,656 cubic yard	= 1/1,728 cu ft	
1 cubic foot (cu ft)	= 1/27 cubic yard	= 1,728 cu in	
1 cubic yard (cu yd)	= 27 cubic feet		
1 teaspoon	= 1/3 tablespoon		
1 tablespoon	= 1/2 fluid ounce	= 3 teaspoons	
1 U.S. fluid ounce (fl oz)	= 1/128 U.S. gallon	= 1/16 U.S. pint	
1 imperial fluid ounce	= 1/160 imperial gallon		
1 gill (gi)	= 1/32 gallon	= 4 fluid ounces	
1 cup	= 1/4 quart	= 1/2 pint	= 8 fluid oz
1 pint (pt)	= 1/8 gallon	= 1/2 quart	= 16 fluid oz
1 quart (qt)	= 1/4 gallon	= 32 fluid ounces	
1 U.S. gallon (gal)	= 231 cubic inches		
1 imperial gallon (gal)	= 277.4 cubic inches		
1 dry pint (dry pt)	= 1/64 bushel	= 1/2 dry quart	
1 dry quart (dry qt)	= 1/32 bushel	= 1/8 peck	
1 peck (pk)	= 1/4 bushel		
1 U.S. bushel (bu)	= 2,150.4 cubic inches		

Conversion Factors for Units of Volume and Capacity

1 cubic inch	= 16.4 cubic centimeters	
1 cubic foot	= 0.0283 cubic meter	
1 cubic yard	= 0.765 cubic meter	
1 fluid ounce	= 29.6 milliliters	
1 U.S. pint	= 0.473 liter	
1 U.S. quart	= 0.946 liter	
1 U.S. gallon	= 0.84 imperial gallon	= 3.8 liters
1 imperial gallon	= 1.2 U.S. gallons	= 4.5 liters
1 dry pint	= 0.55 liters	
1 dry quart	= 1.1 liters	
1 U.S. bushel	= 0.97 imperial bushel	= 35.24 liters
1 imperial bushel	= 1.03 U.S. bushels	= 36.37 liters
1 cubic centimeter	= 0.06 cubic inch	
1 cubic centimeter	= 0.06 cubic inch	
1 cubic meter	= 1.3 cubic yards	
1 milliliter	= 0.034 fluid ounce	
1 liter	= 1.06 U.S. quarts	= 0.9 dry quart
1 dekaliter	= 0.28 U.S. bushel	

STANDARD UNITS OF MEASUREMENT: American, metric, conversion factors

Mass (weight)

1 gram (1 g)	0.035	ounces (oz)
1 kilogram (1 kg)	2.2	pounds (lb)
1 tone (1 t) = (1,000 kg)	1.1	short ton

Temperature

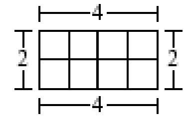
1 degree Celsius (1^{0}C)	degrees C=	5/9(^{0}F-32^{0})
Verification: 0^{0}C= 5/9 (32^{0} F-32^{0})	*100^{0} C=*	*5/9 (212^{0} F^{-}-32^{0})*
1 degree Fahrenheit (1^{0}F)	degrees F=	9/5 (^{0}C)+32^{0}
Verification: 32^{0} F= 5/9 (0^{0} C)+32^{0}	*212 ^{0}F=*	*9/5 (100 ^{0}C)+32^{0}*

The primary (not exclusive) source of information in the preceding tables:

The perimeter of a figure is the length around the edge of the figure. The perimeter is measured in linear units. The word "*units*" used by itself means "linear units". The perimeter of the Figure A at the right is 12 units. The perimeter of a figure is found by adding the lengths of its sides.
$P = S_1 + S_2 + S_3 + S_4 + S_5 \ldots + S_n$ where

S_1 is side 1, S_2 is side 2, and S_n is the last side added.

Figure A



P = 4+2+4+2
P = 12 units
A = 4units • 2units
A = 8 square units

The area of a figure is the number of squares it takes to cover the region within the figure. The area is measured in square units. Since it takes 8 squares to cover the Figure A, the area of the figure is 8 square units.

Concept	Measured in	Example Units
Perimeter 1 dimension	linear units units + units = units	inch (in), foot (ft), yard, mile, millimeter (mm), centimeter (cm), meter (m), kilometer (km)
Area 2 dimensions	square units, or units squared, or unit^2 (unit)(unit) = unit^2	sq in, sq ft, sq yard, sq mile, sq mm, sq cm, sq meter, sq km, ft•ft = sq ft or ft^2 cm•cm = sq cm or cm^2 $3 \bullet 3 = 3^2$; $C \bullet C = C^2$; unit•unit = unit^2

You can find or estimate the area by counting the number of squares within the figure.
Find the perimeter and area of the shaded region.

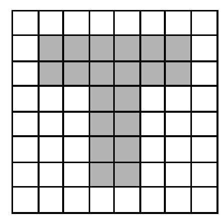


Figure B

Find the perimeter by adding the lengths of the line segments used to draw the figure (the segments that make up the figure).
Perimeter: $P = S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7 + S_8$

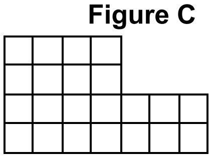
Perimeter: P = 6 + 2 + 2 + 4 + 2 + 4 + 2 + 2 = 24 units, 24 linear units

Find the area by counting the number of squares within the figure.

Area: A = 20 square units

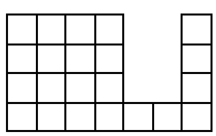
Find the perimeter and area for Figures C, D, E, and F. Write your answers on lined notebook filler paper and then compare your answers with those in the ANSWER SECTION two pages forward.

Figure C



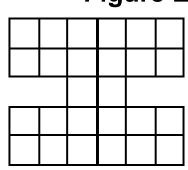
Perimeter =
Area =

Figure D



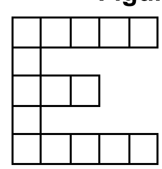
Perimeter =
Area =

Figure E



Perimeter =
Area =

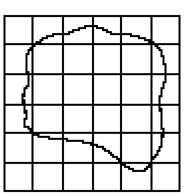
Figure F



Perimeter =
Area =

You can estimate the area of these irregular shapes by counting squares on a grid. Estimate the area for Figures H and I. Write your answers on LNFP and then compare them with those in the answer section.

Figure G

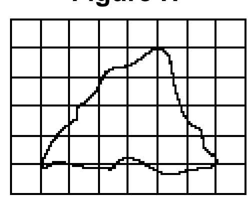


A ≈19 sq units.

The region in Figure G covers 13 full squares. It also covers parts of squares that appear to add about 6 more squares: 13 + 6 = 19

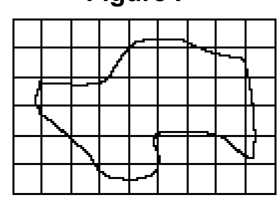
Estimate area for H and I.

Figure H



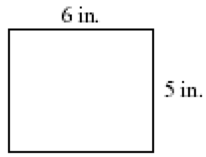
A ≈ .

Figure I



A ≈

You can find the area of a rectangle by multiplying the length times the width.



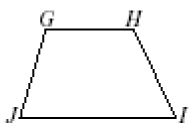
Find the perimeter and area of the rectangle

$P = S_1 + S_2 + S_3 + S_4$ = 6 in. + 5 in. + 6 in. + 5 in. = 22 in.

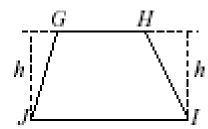
A = wl = (6 in.)(5 in.) = 30 square inches = 30 sq in = 30 in^2

By definition of a rectangle, all the angles in a rectangle are right angles, and therefore, the height is 5 inches.

A polygon is a figure with straight sides. The length of any of the sides of a polygon is called a base. The height is perpendicular to a line containing a base. Since a polygon has several bases, the height will depend on the base you choose. You can find the area of any parallelogram by using A = bh. The height may even be measured outside the figure.

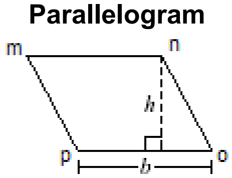


Consider figure GHIJ on the left. *GH* must be extended in order to draw the height. The heights from *J* and *I* are shown; both heights are outside of the figure.



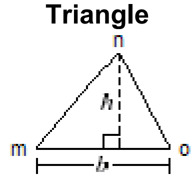
Three common polygons are parallelograms, triangles, and trapezoids. You can use formulas below to find the areas of parallelograms, triangles, and trapezoids. The *base* is represented by the variable *b*. *Height* is represented by the variable *h*.

Parallelogram



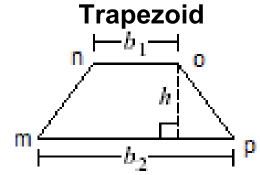
A= bh

Triangle



A= 1/2 bh

Trapezoid



$A = 1/2\,(b_1 + b_2)h$

Find the perimeter and area of each figure.

Parallelogram

mn = po = 11 in

mp = no = 9 in

h = 8 in

$P = S_1 + S_2 + S_3 + S_4$

P=11in+9in+11in+9in

P=40in

A=bh

A=11in•8in

A= 88sq in

Triangle

mn = 10 in

mp = 9 in

b_1 = mo = 11 in

h = 7in

$P = S_1 + S_2 + S_3$

P=10in+9in+11in

P=30in

A=1/2 bh = ½(11in)(7in)

A=38.5 sq in or 38½ sq in

Trapezoid

mn = po = 8 in

b_1 = no = 7 in

b_2 = mp = 17 in

h = 6 in

$P = S_1 + S_2 + S_3 + S_4$

P=8in+7in+8in+17in=40in

$A = 1/2\,(b_1 + b_2)h$

A = 1/2(7in+17in)(6in)

A = ½(24in)(6in)=(12in)(6in)

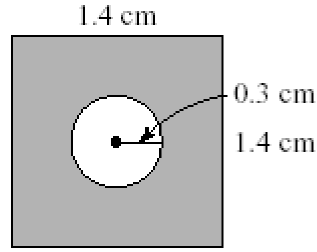
A = 72 square inches

<u>A New Problem Type</u>: find the area of the shaded region.

Sometimes you can find the area of a region by subtracting one area from another.

The formulas for the area and circumference of a circle:
$A = \pi r^2$ **and** $C = \pi d$, **where** r **is the radius and** d **is the diameter.**

Example: Find the area of the shaded region. The area of the shaded region is the area of the square minus the area of the circle.



The substitutions are the same weather you apply
$A = bh - \pi r^2$ **using the general area formula for all parallelograms,** $A = bh$, **or**
$A = S^2 - \pi r^2$ **using the specific area formula for squares,** $A = S^2$
$A \approx (1.4cm)(1.4cm) - (3.14)(0.3cm)^2$
$A \approx 1.96cm^2 - (3.14)(0.09cm^2)$
$A = 1.96cm^2 - 0.2826cm^2 \approx 1.6774cm^2$ **The area of the shaded region is about 1.68** cm^2.

Find the shaded area for Figures J, K, and L. Write your answers on scrape paper and check your answers and check your answers in the ANSWER SECTION.

Figure J

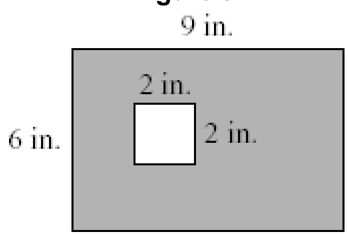


Figure K

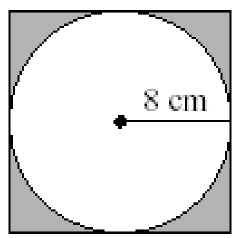
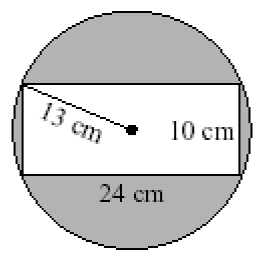


Figure L



Shaded Area ? **Shaded Area ?** **Shaded Area ?**

ANSWER SECTION

Figure C	Figure D	Figure E	Figure F
Perimeter = 7+4+7+4=22units	Perimeter = 7+7+7+7=28units	Perimeter = 10+5+10+5=30units	Perimeter = 11+5+11+5=32units
Area = 22 sq units	Area = 22 sq units	Area = 26 sq units	Area = 15 sq units

Fig H
$A \approx 12$ **sq units**

Fig I
$A \approx 22$ **sq units.**

Figure J
Shaded Area ?
Shaded Area
$A = bh - bh$
$A = (9in)(6in) - (2in)(2in)$
$A = 54in^2 - 2in^2$
$A = 50in$

Figure K
Shaded Area ?
Shaded Area
$A = bh - \pi r^2$
$A = (16cm)(16cm) - (3.14)(8cm)(8cm)$
$A = 256cm^2 - 200.96cm^2$
$A = 55.04cm^2$

Figure L
Shaded Area ?
Shaded Area
$A = \pi r^2 - bh$
$A = (3.14)(13cm)(13cm) - (10cm)(24cm)$
$A = 530.66cm^2 - 240cm^2$
$A = 290.66cm^2$

This is an adaptation of the original material: AWSM Foundations of Algebra and Geometry

Putting it all together:

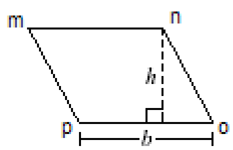
Consider Shape

↙ ↘

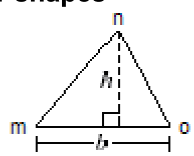
Situation 1: Regular shape

↓

Example regular shapes



parallelogram



triangle

↓

pick formula in provided table

↓

write the formula

↓

substitute numbers and units into the formula

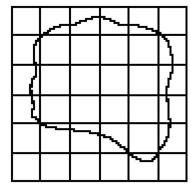
↓

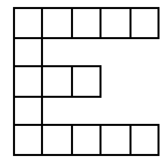
calculate and show answer with correct units:
area in square units;
perimeter & circumference in (linear) units

Sit 2: Irregular shape (or a shape with a grid)

↓

Example irregular shape and a shape with a grid.





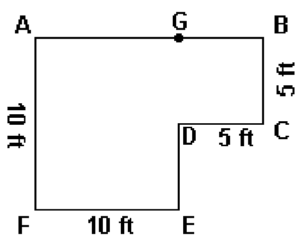
has no formula

↓

Area → count or estimate the squares
A = give area in square units
Example A = 12 square feet
Perimeter → count the (linear) units (lengths) around the edge
P = give perimeter in units (linear units)
Example P = 15 feet

**** Look for formula in the provided (see next page) Reference Sheet ; if present, use its formula.**

Situation 3: Some irregular shapes can or must be separated into several regular shapes so you have enough information to find the area or the perimeter.



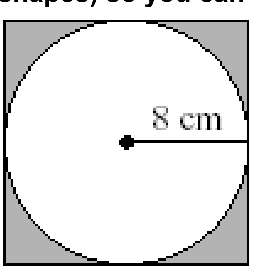
What is the area and perimeter of figure ABCDEF?
1 - First figure out what regular shapes makeup the whole figure. The shape to the left is not a regular shape, but it can be separated into two regular shapes: a 10ft-by-10ft square and a 5ft-by-5ft square.

2- Second figure out the missing measurements that you need and neatly add those measurements to the figure.
If EF = 10ft and CD = 5ft, then AB = 15ft.
If AF = 10ft and BC = 5ft, then DE = 5ft.
Shape's area = area of the 10ft square plus the area of the 5ft square.

$A = b_1h_1 + b_2h_2$ = (10ft)(10ft) + (5ft)(5ft) = 100sq ft +25sq ft = 125sq ft

$P=S_1+S_2+S_3+S_4+S_5+S_6$ = 15ft + 5ft + 5ft + 5ft + 10ft + 10ft = 50ft (I went clockwise.)

Situation 4: Some shapes (like the shaded area below) must be separated into several parts (shapes) so you can find the answer(s).



What is the shaded area?

First, figure out what regular shapes makeup the whole figure. The shape to the left is not a regular shape, but it can be separated into two regular shapes: a 16cm-by-16cm square and a circle with a radius of 8 cm.
Shaded area = $bh - \pi r^2$ = (16cm)(16cm) – (3.14)(8cm)(8cm)
Shaded area = 254 sq cm – 200.96 sq cm = 53.04 sq cm
or Shaded area = 254 cm^2 – 200.96 cm^2 = 53.04 cm^2

<u>Note</u> that cm is squared (cm^2) and not 53.04 because, 53.04^2 = 2,813.2416

Situation 5: There are numerous other situations, too many to list individually, but many are combinations of situations 1-4 above.

The table below is comparable (much like) to the one provided for tests and assignments.

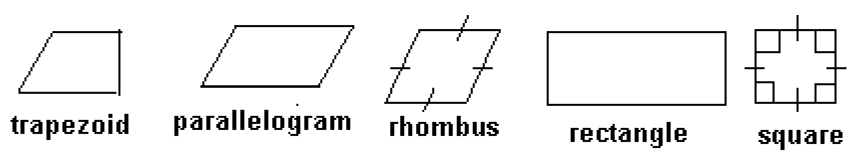
Use it for all perimeter, area, and volume test questions and exercises (in textbook and handouts), and familiarize yourself with the table's contents and organization.

Junior High Reference Sheet

(Figure)	Perimeter	Area
Parallelogram	$P=S_1+S_2+S_3+S_4$	$A=b\bullet h$
Rectangle	$P=S_1+S_2+S_3+S_4$ or $P=2l+2w$	$A=b\bullet h$
Square	$P=4S$	S^2
Triangle	$P=S_1+S_2+S_3$	$A=½\, b\bullet h$
Trapezoid	$P=S_1+S_2+S_3+S_4$	$A=½\,(b_1+b_2)h$
Circle	**Circumference** $C=2\pi r$	$A=\pi r^2$
		Volume
Rectangular prism		$V=lwh$
Rectangular pyramid		$V=1/3\ lwh$
Cylinder		$V=\pi r^2 h$
Cone		$V=1/3\ \pi r^2 h$
Sphere		$V\text{-}=4/3\ \pi r^2$
b = base h = height	**Key** l = length w = width s = side	r = radius d = diameter Use 3.14 for π

Using this Reference Sheet for test successfully requires your understanding and recalling certain facts and concepts at test time; some follow.

quadrilaterals:



Regular polygons (triangles, quadrilaterals, rectangles, pentagons, hexagons, etc.) are polygons that are both equilateral and equiangular (have congruent sides and congruent angles.)

quadrilateral – polygon with four sides
rectangle: a quadrilateral with four right angles

)

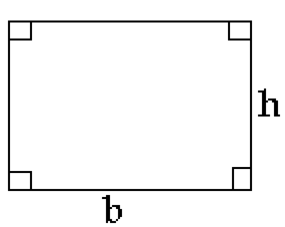


Figure 1

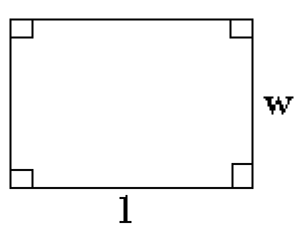
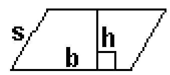


Figure 2

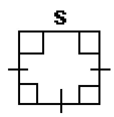
Area = (base)(height) $A_R = bh$
or
Area = (width)(length) $A_R = wl$

Perimeter = $P_R = 2b + 2h$
or
Perimeter = $P_R = 2l + 2w$

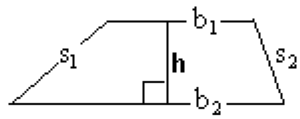
If the plane of the rectangle is vertical, we commonly call the dimensions "base" and "height."
If the plane of the rectangle is horizontal, we commonly call the dimensions "length" and "width."



parallelogram
Figure 3

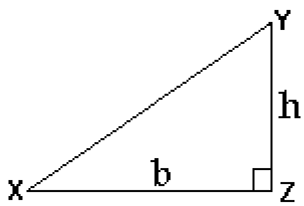


square
Figure 4

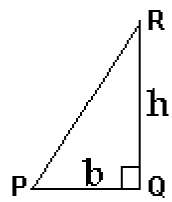


trapezoid
Figure 5

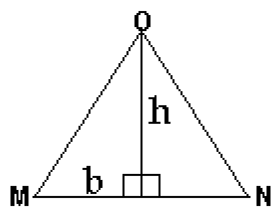
Area $_{\text{(parallelogram)}}$ = (base)(height) = bh
Area $_{\text{(square)}}$ = (side)(side) = s^2
Area $_{\text{(trapezoid)}}$ = ½ (height)(base_1+ base_2)
= ½ h(b_1 + b_2)
Perimeter $_{\text{(parallelogram)}}$ = 2b + 2s
Perimeter $_{\text{(square)}}$ = 4s
Perimeter $_{\text{(trapezoid)}}$ = $b_1 + b_2 + s_1 + s_2$



ΔXYZ
Figure 6



ΔPQR
Figure 7

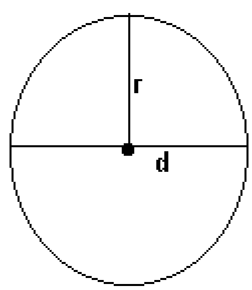


ΔMNO
Figure 8

Area = (base)(height) = A_T = ½ bh

Perimeter = $P_T = S_1 + S_2 + S_3$

For ΔMNO, the sides are:
$\overline{MN}$, $\overline{NO}$, and $\overline{MO}$



diameter, d = 2r where r is the radius

circumference, C, is the distance around a circle (in linear units: feet, inches, miles, centimeters, meters.)

C = πd = 2πr where r is the only variable and is given in linear units.

Area $_{\text{(cicle)}}$ = πr^2 where r is the only variable and is given in linear units; however, (r)(r) = the product, r^2, in sq units.

I'm just using the space for review!

MULTIPLICATION of decimal numbers

1. Write the numbers and multiply exactly like the multiplication of whole numbers disregarding decimal placement. Place the numbers in straight, neat lines; the decimal points do not have to be arranged in any way.

2. After multiplying, find the sum of digits right of the decimal point in both the multiplicand (top number) and multiplier (bottom number), and counting from right to left, count off that total number of digits in the product.

4.6	(1) (1 digit)
X .04	+ (2) (2 digits)
.184	(3) (3 digits right of the decimal)

3. Zeros may need to be supplied if there are not enough numerals (place values) in the product.

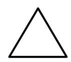
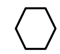
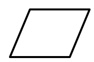
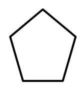
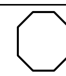
46	a whole number	(0) 46 is	.46	forty six hundredths	(2)
X 4	a whole number	+ (0) 4 is	X .04	four hundredths	+ (2)
184	a whole number	(0) §§	.0184	one hundred eighty-four ten thousandths	(4) §§

§§ Sum of the digits right of the decimal in both the multiplicand and multiplier.

<u>bounded plane</u> (a plane with boundaries) is a two-dimensional and the sides are connected (closed); it has both breadth and height but no thickness, but no thickness (depth) because a plane is "flat."

<u>plane figures</u> (shapes) are closed (connected) sets of points in a plane that have an interior and an exterior (an inside and a outside.) They have breadth (width) and height, but no depth (thickness); therefore, they have two dimensions, 2-D, and surface areas that can be measured in square units.

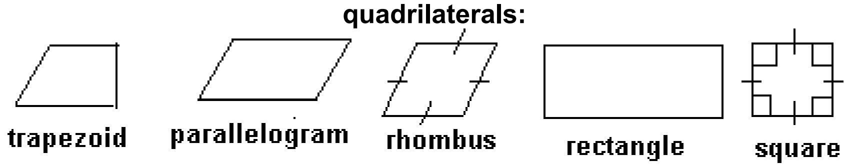
A. circle - A plane curve everywhere equidistant from a given fixed point, the center.
B. polygon - A <u>closed</u> plane figure bounded by three or more line segments.

Number of sides	Polygon	Example shape	Number of sides	Polygon	Example shape
3	triangle tri = three		6	hexagon hex = six	
4	quadrilateral quad = four		7	heptagon hept = seven	
5	pentagon pent = five		8	Octagon Oct = eight	

Regular polygons (triangles, quadrilaterals, rectangles, pentagons, hexagons, etc.) are polygons that are both equilateral and equiangular: have congruent (equal lengthed) sides and congruent (equal) angles.

quadrilateral – polygon with four sides
rectangle: a quadrilateral with four right angles

quadrilaterals:



perimeter – the distance around a figure (in linear units: feet, inches, miles, centimeters, meters)
area – the number of square units in a plane (sq feet, sq inches, sq miles, sq cm, sq meters)
to calculate area, the linear units must be the same before you multiply (units)(units) = (sq units)

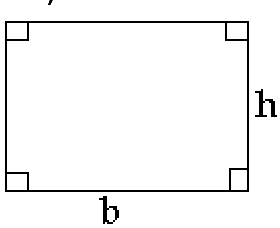


Figure 1

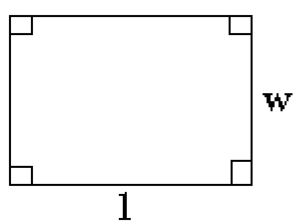


Figure 2

Area = (base)(height) $A_R = bh$
or
Area = (width)(length) $A_R = wl$

Perimeter = $P_R = 2b + 2h$
or
Perimeter = $P_R = 2l + 2w$

If the plane of the rectangle is vertical, we commonly call the dimensions "base" and "height."
If the plane of the rectangle is horizontal, we commonly call the dimensions "length" and "width."

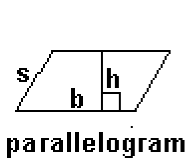


Figure 3

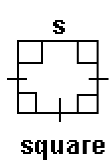


Figure 4

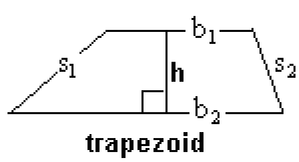
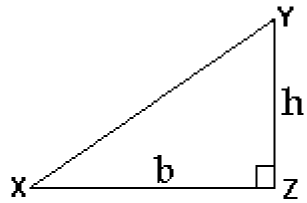
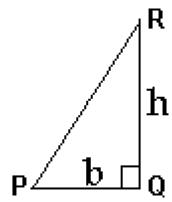


Figure 5

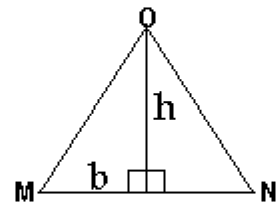
Area $_{(parallelogram)}$ = (base)(height) = bh
Area $_{(square)}$ = (side)(side) = s^2
Area $_{(trapezoid)}$ = ½ (height)($base_1$+ $base_2$)
= ½ $h(b_1 + b_2)$
Perimeter $_{(parallelogram)}$ = 2b + 2s
Perimeter $_{(square)}$ = 4s
Perimeter $_{(trapezoid)}$ = $b_1 + b_2 + s_1 + s_2$



ΔXYZ
Figure 6



ΔPQR
Figure 7

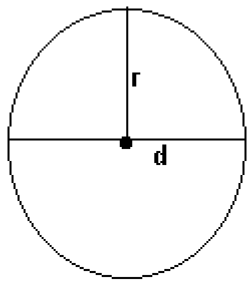


ΔMNO
Figure 8

Area = (base)(height) = A_T = ½ bh

Perimeter = $P_T = S_1 + S_2 + S_3$

For ΔMNO, the sides are: $\overline{MN}$, $\overline{NO}$, and $\overline{MO}$



diameter, d = 2r where r is the radius

circumference, C, is the distance around a circle (in linear units: feet, inches, miles, centimeters, meters.)

C = πd = $2\pi r$ where r is the only variable and is given in linear units.

Area $_{(circle)}$ = πr^2 where r is the only variable and is given in linear units; however, (r)(r) = the product, r^2, in sq units.

<u>OUTLINE OF GEOMETRIC FIGURES</u>: this is only representative list of figures.

I. PLANE FIGURES - a closed (connected) set of points in a plane that has an interior and an exterior (an inside and a outside); a plane figure has breadth (width) and height, but no depth (thickness); therefore it has two dimensions, 2-D, and surface area which can be measured.

- **A. circle - A plane curve everywhere equidistant from a given fixed point, the center.**
- **B. polygon - A closed plane figure bounded by three or more line segments.**
  - **1. triangle - a three-sided polygon.**
    - **a. scalene triangle - a triangle in which no two sides are congruent**
    - **b. isosceles triangle - a triangle with two congruent sides**
    - **c. equilateral triangle - a triangle with three congruent sides**
    - **d. right triangle - a triangle that one angle that measures 90 degrees and therefore the sum of the other angles is 90 degrees.**
    - **e. acute triangle - a triangle with three acute angles (all less than 90 degrees)**
    - **f. obtuse triangle - a triangle with one obtuse angle and two acute angles.**
  - **2. quadrilateral - a four-sided polygon**
    - **a. trapezoid - a quadrilateral with one pair of parallel sides**
    - **b. parallelogram - a quadrilateral whose opposite sides are parallel**
      - **(1) rhombus - a parallelogram whose sides are congruent; an equilateral parallelogram**
      - **(2) rectangle - a parallelogram with four right angles**
        - **(a) square - a <u>rectangle</u> with four congruent sides AND a <u>rhombus</u> with four right angles.**
  - **3. pentagon - a five-sided polygon**
  - **4. hexagon - a six-sided polygon**
  - **5. heptagon - a seven-sided polygon**
  - **6. octagon - an eight-sided polygon**
  - **7. N-agon - an N-sided polygon**

II. SPACE FIGURE - also called a solid, is a three-dimensional geometric figure, or portion of space, that has breadth (width) and height, and depth (thickness); therefore, it has three dimensions, 3-D, and a volume which can be measured. (Space - extends in all directions without bounds (limits) and contains all points, lines, and planes.)

- **A. sphere - A solid that has all surface points equidistant from a fixed point.**
- **B. cylinder - A solid bounded by two parallel planes and such a surface, especially such a surface having a circle**
- **C. cone**
- **D. polyhedron - A solid bounded by polygons.**
  - **1. prism - A solid figure whose bases or ends have the same size and shape and are parallel to one another, and each of whose sides is a parallelogram.**
    - **a. triangular prism**
    - **b. rectangular prism**
    - **c. square prism**
    - **d. N-gular prisms**
  - **2. pyramid - A solid figure with a polygonal base and triangular faces that meet at a common point.**
    - **a. triangular pyramid**
    - **b. rectangular pyramid**
    - **c. square pyramid**
    - **d. N-gular pyramid**

Concepts

geometry (n) a branch of mathematics that deals with the measurement of the shape, size, relations and other properties of points, lines, angles, planes, and space figures

arithmetic (a-RITH-me-tic) - (n) a branch of mathematics that usually deals with nonnegative numbers and the application of the operations of addition, subtraction, multiplication, and division.

geometric (adjective) of, about, or pertaining to geometry

arithmetic (ar-ith-MET-ic) - (adjective) of, about, or pertaining to a-RITH-me-tic.

figure (n) as a general term, a figure is the external shape or outline of something.

arithmetic figure (n) a symbol for a whole number such as "3", "4", and "23".

geometric figure (n) in geometry, a figure is a closed shape; a closed (connected) set of points that has an interior and an exterior (an inside and a outside.) Examples include triangle, square, circle, and cylinder. **According to this definition, a line is not a figure, but sometimes a line is referred to as a figure.

dimension (n) the measurement in a straight line of the breadth, height, or depth (width, height, and/or thickness) of something. **If an object has more than one dimension, then each measurement must be at a right angle (90 degree angle) to measurements already taken.

continued

point (n) is a location (having a specific position), but having no dimension, 0-D, (no breadth, height, or thickness.) We represent (draw a model of) a point with a dot. We can see our representation (the dot), but since in reality a point is a location, it cannot be seen.

line (n) contains more than one point on a straight path; extends in opposite directions indefinitely (forever.) **A line is a concept (a mental construction), and, therefore, cannot be seen. We CAN represent (draw a model of) a SEGMENT of a line and identify (name) it by two points on the representation (model.) A line containing points A and B can be named line AB

collinear points - (n) points on the same line.

noncollinear points - (n) points not on the same line.

linear (adjective) means alone a line or related to a line

linear units (n) standard units to measure the distance between two points; e.g. feet, inches, miles, meters, centimeters (cm), and kilometers

segment (or line segment) (n) a part of a line consisting of two endpoints and all the points between them. Since a segment has endpoints, it has breadth, but no height or thickness; therefore a segment has one dimension, 1-D. A segment containing endpoints X and Y can be identified/named segment XY

parallel lines (n) lines that do not intersect no matter how far extended and, by definition, are equal distance apart.

perpendicular lines - (n) lines that intersect and form right angles.

plane - (n) is everywhere flat, it extends indefinitely in all directions "on" it; a plane is a concept (a mental construction), and, therefore, cannot be seen. We CAN represent (draw a model of) a plane and identify/name it plane P, plane T, and so on. The position of a plane is determined by any one of the following three conditions:
(1) three points not on a line, (2) a line and a point not on that line, (3) two intersecting lines, OR (4) two parallel lines.

coplanar - (adjective) angles, lines, points in the same plain

congruent - (adjective) having the same size and shape
congruent angles - angles with the same measure
congruent lines - lines with the same measure

linear points - (n) points on the same line or of the same line.
noncollinear points - (n) points not on the same line.

angle - (n) (you know what an angle) here it is two noncollinear rays with a common end point

$\angle$m - represents measure angle m

acute angle - angle of with degree measure m such that Ø < m < 90;
$\angle$m is between 0° and 90°
obtuse angle - angle of with degree measure m such that 90 < m < 180.
$\angle$m is between 90° and 180°
right angle - angle of with degree measure m = 90
straight angle - angle of with degree measure m = 180

<u>EXPANDED OUTLINE OF GEOMETRIC FIGURES</u>

Note: Geometric Figures listed here are only a few of the figures: those that you be able to identify and you should be able to compute their area and/or volume.

I. <u>PLANE FIGURES</u> - a closed (connected) set of points in a plane that has an interior and an exterior (an inside and a outside); a plane figure has breadth (width) and height, but no depth (thickness); therefore it has two dimensions, 2-D, and surface area which can be measured.

A. <u>circle</u> a set of points in a plane that are a given distance from a point in the plane; a closed curve in a plane, all whose points are the same distance from a point called the enter.

B. <u>polygon</u> - a closed figure made up of segments in a plane; (advanced) the union of three or more coplanar segments such that the segments intersect only at endpoints, each endpoint belongs to exactly two segments, and no two segments with a common endpoint are collinear.

1. <u>triangle</u> - a three-sided polygon; (advanced) the union of three line segments determined by three noncollinear points.
- **a. <u>scalene</u> triangle - a triangle in which no two sides are congruent**
- **b. <u>isosceles</u> triangle - a triangle with two congruent sides**
- **c. <u>equilateral</u> triangle - a triangle with three congruent sides**
- **d. <u>right</u> triangle - a triangle that one angle that measures 90 degrees and therefore the sum of the other angles is 90 degrees.**
- **e. acute triangle - a triangle with three acute angles (all less than 90 degrees)**
- **f. <u>obtuse</u> triangle - a triangle with one obtuse angle and two acute angles.**

2. <u>quadrilateral</u> - a four-sided polygon
- **a. <u>trapezoid</u> - a quadrilateral with one pair of parallel sides**
- **b. <u>parallelogram</u> - a quadrilateral whose opposite sides are parallel**
  - **(1) <u>rhombus</u> - a parallelogram whose sides are congruent (opposite angle would be congruent, too)**
    - **(a) <u>square</u> - a rectangle with four congruent sides AND a rhombus with four right angles.**
  - **(2) <u>rectangle</u> - a parallelogram with four right angles**
    - **(a) <u>square</u> (again)- a rectangle with four congruent sides AND a rhombus with four right angles.**

3. <u>pentagon</u> - a five-sided polygon
4. <u>hexagon</u> - a six-sided polygon
5. <u>heptagon</u> - a seven-sided polygon
6. <u>octagon</u> - an eight-sided polygon
7. <u>N-agon</u> - an N-sided polygon

<u>space</u> - extends in all directions without bounds (limits) and contains all points, lines, and planes. Space is a concept (a mental construction) and, therefore cannot be seen.

II. SPACE FIGURE - also called a solid, is a three-dimensional geometric figure, or portion of space, that has breadth (width) and height, and depth (thickness); therefore it has two dimensions, 3-D, and volume which can be measured.

A. sphere - the set of all points that are equal distance from a given point

B. cylinder - (right, circular cylinder) - (1) a 3-dimensional figure made up of two congruent circles in two parallel planes, their interiors, and all segments having end points in each circle and parallel to the segment (central axis.) Each circle and its interior is a base of the cylinder. (2) the set of endpoints of all equal-lengthed (line) segments that are perpendicular to and equal distance from a (central axis) segment: one endpoint of each radiating segment is on the central axis and the other endpoint is one of the set of endpoints making up the cylinder

1. oblique, circular cylinder - a cylinder whose axis is not perpendicular to the base(s)

C. cone - (right, circular cone) - a 3-dimensional figure made up of a circle, its interior, a given point (called a vertex) and all segments having endpoints at the vertex and a point in the circle.

D. polyhedron

1. prism
    - a. triangular prism
    - b. rectangular prism
    - c. square prism
    - d. N-gular prisms
2. pyramid
    - a. triangular pyramid
    - b. rectangular pyramid
    - c. square pyramid
    - d. N-gular pyramid

- Put your name, today's date, and class period in the upper right corner and put the title of the assignment on the first line of the sheet as shown below.

(top of page)

Jose Garcia
August 14, 02
1st Period

❍ *Textbook, pp. 20-24, Exercises 10-17*

- Do all work assigned in room 112 in erasable black lead.
- Unless otherwise announced, do assignments on LNFP: standard size, lined, notebook, filler paper (8.5 in by 11.0 in.)
- Block print numbers and letters (not stylized.)
- Form (make) easily-to-read characters (numbers and letter) : 4's don't look like 9's. If you need to, practice your writing your characters.
- Circle your problem numbers.
- Skip a line between problems, and separate one problem from another with left-to-right space.
- Work each problem down the page (if amendable.)
- Show what know. Simply giving the answer to a problem is <u>usually</u> not enough. (Most odd answers are in the back of your textbook.) Show all work (setup and steps) according to the examples given by your teachers here in room 112. Each time you perform an operation (add, subtract, multiply, or divide), it should be on a separate line.

See the example expression evaluated and equation solved below.

Evaluate the expression: 4 + 3 • 7.
The first operation (3 times 7) requires its own step (on line 2), and the second operation (4 plus 21) requires its own step.

Solve the equation 3y + 4 = 25

① 4 + 3 • 7 = E
4 + 2 1 = E
2 5 = E

E represents some numb er

②
$$3y + 4 = 25$$
$$3y + 4 - 4 = 25 - 4$$
$$3y = 21$$
$$\frac{3y}{3} = \frac{21}{3}$$
$$y = 7$$

(1) By following these procedures, you will have a better visual image of the problem, and you are more likely to get correct answers!
(2) Your teachers want to know if you use the proper steps to reach your answer.

- <u>Be</u> <u>neat</u>! <u>Don't</u> <u>scribble</u> on your paper. The appearance of your work sends a message, too.

- Always check your own work not less than every (1) ten minutes or more often on something new (2) at the end of each page and (3) at the end of each activity. <u>Lightly</u> place a 'X' by every wrong answer, and optionally put a check mark √ or a capital 'C' by each correct answer. Then go back to your seat and redo problems and questions answered wrong. Remove the 'X' from corrected answers. Do not copy answers, only check by writing a "√ " , "C" or "X").

September 9, 2002

absolute value - (n) A number's distance from zero on a number line. The absolute value of –4 is 4; the absolute value of 4 is 4.

algebra - (n) a method of solving practical problems by using symbols, usually letters, for unknown quantities; a advanced arithmetic in which symbols, usually letters of the alphabet, represent numbers or members of a specified set of numbers and are related by operations that hold for all numbers in the set.

algebraic method - (n) The use of symbols to represent quantities and signs to represent their relationships.

algebraic sentence - (n) A general term for equations and inequalities.

algorithm - (n) A mechanical procedure for performing a given calculation or solving a problem through step-by step procedures such as those used in long division.

angle measure - (n) The measure of the space between two lines that meet in a point. Angles are measured in degrees or radians.

axiomatic systems - (n) Systems that include self-evident truths; truths without proof and from which further statements, or theorems, can be derived.

base and exponent - (nouns) In 3^4, 3 is the base, and 4 is the exponent. The exponent tells how many times the base is multiplied by itself to attain a regular (natural) number.

Example $3^4 = 3 \bullet 3 \bullet 3 \bullet 3 = 81$

coefficient - (n) the number 5 is the coefficient of X in the term 5X; a coefficient is a constant.

combine - (v) like terms can be combined; unlike terms cannot be combined. See "like terms" and "unlike terms"

Example: $9X^2 + X + 3 + 4X + 5$ can be combined to $9X^2 + 5X + 8$

equation - (n) A mathematical statement in which one expression is equal to another.

evaluate - (v) or find the value of an expression means to find its value when numbers are substituted for variables (letters) in the expression

What is the value of (X - 3), if X =7

If you substitute 7 for X, then (7 - 3) = 4

exponent - (n) Tells how many times a number or variable is used as a factor. For example, 6 with an exponent of 3 (6^3) indicates that 6 is a factor 3 times (6 x 6 x 6).

exponential function - (n) A function commonly used to study growth and decay. It has a form $y = a^x$.

expression - (n) A mathematical phrase with no equal sign, such as $3x$, 6, $2n + 3m$.

factor - (n) `Any of two or more quantities that are multiplied together. In multiplication, each number multiplied is a factor of the product. Then a number is a factor of a given number if the given number is divisible by the number.

Example: 2 and 3 are factors of 6.

Identify - (v) To state, match, select, write.

imaginary numbers - (n) The square root of a negative number usually expressed using *i*.

integers- (n) A set of numbers consisting of the whole numbers and their opposites {. . . –2, -1, 0, 1, 2 . . .}.

irrational numbers - (n) A set of numbers that cannot be represented as an exact ratio of two integers. For example, the square root of 2.

like terms - (n) like terms (1) contain the same variable (letter) or variables and (2) variables are raised to the same power; 2X and 5X are like terms; 5Xy and 3Xy are like terms; 8b and $12b^2$ are unlike terms because "b" is not raised to the same power. Power is defined and described at the top of the next page.

multiple - (n) A number into which another number may be divided with no remainder.

operation - (n) (mathematical operation) - add, subtract, multiply and divide are mathematical operations represented by symbols +, -, X, and ÷

monomial - (n) In algebra, an expression consisting of a single term such as 5*y*.

binomial - (n) In algebra, an expression consisting of two terms connected by a plus or minus sign, such as 4*a* + 6.

polynomial - (n) In algebra, an expression consisting of two or more terms such as $x^2 - 2xy + y^2$.

power - 3^4 means 3 raised to the fourth power or the fourth power of 3; same as example immediately above. In this sense, "power" and "exponent" mean the same thing.

primes - (n) Counting numbers that can only be evenly divided by two numbers which are the number itself and 1. For example, the numbers 2, 3, 5, 7.

proportion - (n) An equality between ratios. For example, 2/6 = 3/9.

ratio - (n) A comparison expressed as indicated division. For example, there is a ratio of three boys to two girls in our class (3/2, 3:2).

rational numbers - (n) Numbers that can be expressed as an exact ratio of two integers.

real numbers - (n) All rational and irrational numbers.

rearrange - (v) in an expression like 9X + 8 - 12X + 7, we can rearrange the terms to simplify:

9X + 8 - 12X + 7 is rearranged as (9X - 12X) + (8+7) to simplify to -3X + 15

sentence - (n) a mathematical sentence contains either = , < , > , ≤ , ≥

simplify - (v) to simplify an expression means to replace it with the least complicated equivalent expression
Example: 5X - 3X + 7 is simplified to 2X + 7

solve - (v) to solve an equation is to find all its solutions, the values of each variable (variables are usually represented by letters)

satisfy - (v) if a variable in an equation is replaced with a number, and the resulting statement is true, then the number satisfies the equation

term - (n) in the equation, 2X + 3Y - 25 = 0 , the terms are 2X, 3Y, 25, AND 0. Terms are usually separated by addition and/or subtraction operations symbols (+, -) and the equals symbol (=) Terms can be separated by < , > , ≤ , ≥ as well as =, +, AND -

unlike terms - (n) In the expression, 6x and 3y, the terms are unlike because the variables (X and Y) are not the same. 6x and $6x^2$ are unlike terms because the variable x is not raised to the same power.

variable - (n) a symbol, usually a letter, used to represent a number; a place holder (positioned) in an algebraic expression. In 3*x* + *y* = 23, *x* and *y* are variables.

whole numbers - (n) The counting numbers and zero {0, 1, 2, 3 . . .}.

The first table given here is a quick reference to aid you when working problems or grading tests.

X	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2	3	4	5	6	7	8	9	10	11	12	13
2	2	4	6	8	10	12	14	16	18	20	22	24	26
3	3	6	9	12	15	18	21	24	27	30	33	36	39
4	4	8	12	16	20	24	28	32	36	40	44	48	52
5	5	10	15	20	25	30	35	40	45	50	55	60	65
6	6	12	18	24	30	36	42	48	54	60	66	72	78
7	7	14	21	28	35	42	49	56	63	70	77	84	91
8	8	16	24	32	40	48	56	64	72	80	88	96	104
9	9	18	27	36	45	54	63	72	81	90	99	108	117
10	10	20	30	40	50	60	70	80	90	100	110	120	130
11	11	22	33	44	55	66	77	88	99	110	121	132	143
12	12	24	36	48	60	72	84	96	108	120	132	144	156
13	13	26	39	52	65	78	91	104	117	130	143	156	169

The second table here gives the multiplication facts that you should know without hesitation.
Objective (goal): Give the 103 responses (multiplication facts) in 7 minutes with 100% accuracy.

X	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2	3	4	5	6	7	8	9	10			
2	2	4	6	8	10	12	14	16	18	20			
3	3	6	9	12	15	18	21	24	27	30			
4	4	8	12	16	20	24	28	32	36	40			
5	5	10	15	20	25	30	35	40	45	50			
6	6	12	18	24	30	36	42	48	54	60			
7	7	14	21	28	35	42	49	56	63	70			
8	8	16	24	32	40	48	56	64	72	80			
9	9	18	27	36	45	54	63	72	81	90			
10	10	20	30	40	50	60	70	80	90	100			
11											121		
12												144	
13													169

Circle day of week
Name (Print)___________________________________ Period ___ Qtr ___ Date ___________ Day M T W Th F

Minutes allowed:______ Number correct:______ Number wrong (incorrect & incomplete):______

Objective (goal): Complete the 103 responses (multiplication facts) in 7 minutes with 100% accuracy.

INSTRUCTIONS: NR = not required

Initial Testing Instructions (for the first several times you do this activity):
You have 12 minutes to complete as many responses as you can. After the timed session, draw a light (not dark) line through those cells (boxes) that you did not complete. Then follow the grading instructions below. Finally, use a printed table to correct and complete your learning table.

Proficiency Testing Instructions:
You have 8 minutes. Complete the table as quickly as possible. As you complete the table, circle the responses (multiplication facts) that you hesitated on. Complete the grading instructions below. Then use a printed table to correct and complete your learning table.

Grading Instructions: Check your own work or exchange papers as directed. First draw a line through every cell (box) with no answer, and then put a check mark in every cell with the wrong answer. Put minutes allowed, the number correct responses and the number wrong (incorrect plus incomplete) responses in the blanks at the top of the page. There are 103 responses.

X	1	2	3	4	5	6	7	8	9	10	11	12	13
1											NR	NR	NR
2											NR	NR	NR
3											NR	NR	NR
4											NR	NR	NR
5											NR	NR	NR
6											NR	NR	NR
7											NR	NR	NR
8											NR	NR	NR
9											NR	NR	NR
10											NR	NR	NR
11	NR	NR	NR	NR	NR	NR	NR	NR	NR	NR		NR	NR
12	NR	NR	NR	NR	NR	NR	NR	NR	NR	NR	NR		NR
13	NR	NR	NR	NR	NR	NR	NR	NR	NR	NR	NR	NR	

____ ____
(Signature) I have neither given nor received aid on this test. (Print name of student grader) I have not given aid on this test.

Most if the modern alphabets in modern Europe are modeled after the Greek alphabet, which in turn was modeled from the Phoenician alphabet about 1000 B.C. There are two forms of the Greek alphabet: eastern and western. The Roman alphabet was modeled after the western Greek form, and in turn, the English alphabet. The eastern form of the alphabet came to be used in ancient and modern Greece and the model of the Cyrillic alphabet, used in Russian, Bulgarian, and Serbian languages.

Greek letters

Upper case	Lower case	Letter's name	English sound
Α	a	alpha	a as in arm
Β	β	beta	b as in but
Γ	y	gamma	g as in get
Δ	Δ	delta	d as in do
Ε	e	epsilon	e as in held
Ζ	ζ	zeta	z as in adze
Η	η	eta	e as in they
Θ	θ	theta	h as in thin
Ι	ι	iota	i as in machine
Κ	κ	kappa	k as in kite
Λ	λ	lambda	l as in lamb
M	μ	mu	m as in man
N	ν	nu	n as in now
Ξ	ξ	xi	x as in ax
O	o	omicrom	o as in for
Π	π	pi	p as in pie
Ρ	ρ	rho	r as in ran
Σ	σ, s	sigma	s as in sat
Τ	τ	tau	t as in tar
Y	υ	upsilon	u as in rude
Φ	φ	phi	f as in fill
Χ	x	chi	k as in elkhorn
ψ	ψ	psi	s as in upset
Ω	ω	omega	o as in hold

English sounds from World Book, "G", Volume 8, page 371, © 1978

Arabic number symbols, 0 through 9, are used in our numeration system (counting system) today. A numeral is a single symbol or a collection of symbols (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) that designate (represent) a particular number (value). The numeration system (counting system) used in ancient Rome involved using a single letter or combination of letters to represent a particular number (value); Roman numbers are primarily used in outlines today.

Arabic Number	Roman Number	Arabic Number	Roman Number
1	I	21	XXI
2	II	29	XXIX
3	III	30	XXX
4	IV	40	XL
5	V	48	XLVIII
6	VI	49	IL
7	VII	50	L
8	VIII	60	LX
9	IX	90	XC
10	X	98	XCVIII
11	XI	99	IC
12	XII	100	C
13	XIII	101	CI
14	XIV	200	CC
15	XV	500	D
16	XVI	600	DC
17	XVII	900	CM
18	XVIII	1,000	M
19	XIX	1,666	MDCLXVI
20	XX	1,970	MCMLXX

Except for adjectives in parenthesis, all words are nouns.

RANGE, amount, ambit, compass (space within), extension, extent, magnitude, orbit, purview, range, reach, realm, scope, span, stretch,

QUANTITY, (all-purpose) amount, extent, measurement, intensity, amplitude, frequency, magnitude, level, pitch

QUANTITY, (absolute, complete) amount, sum, size, valuation, value, cost, charge, measurement

QUANTITY, (relative) degree, proportion, ratio, scale, comparison, percentage, percent, percentile, PORTION, PART, share, ration

MEASUREMENT, dimension, width, length, depth, altitude, height, amplitude, frequency, level, pitch,

Rate, tenor, way, speed, MOTION

Gradualism, gradualness, evolution, SLOWNESS, increment, pace, rate, SPEED

Key, register, (musical) NOTE

Gradation, graduation, calibration, DIFFERENTIATION

Change, difference, differential, gap, shade, hint, nuance, grade, level, stepping-stone, step, rung, stair, rise, fall, climb, ascent, descent

Point, stage, MILESTONE, turning point, crisis, JUNCTURE

Mark, peg, notch, score, INDICATOR

Bar, line, interval, NOTATION

ARRANGEMENT, place, placement, position, order, level, rank, grade, CLASSIFICATION, kind, class, kind, SORT

Standard, rank, grade, (serial) PLACE

Sphere, station, status, (social) class, caste, standing, footing, CIRCUMSTANCE

OFFICE, (ecclesiastical) rank, (military) rank, lieutenancy, captaincy, majority, colonelcy

Hierarchy, AUTHORITY

PROGRESSION, (arithmetical) progression, (geometrical) progression, (harmonic) progression

Trigonometric ratios: sine, tangent, secant, cosine, cotangent, cosecant

Content was modified for classroom use by T. Grimes.

Suggestions and corrections

There have been errors, typos, etc. in previous versions; I expect there are some here, too. Please send suggested improvements or description of errors; do include page number, topic, and junior high or high school guide, etc.

Sent your comments/suggestions/corrections to:

mathguide@grimesez.com

Unless you request otherwise, your name will appear here as a user-editor in future updated versions.

Updated versions

The latest version will be available at *grimesez.com* and/or *grimestech.com*

T L Grimes: Prologue and *postscript*

The genesis of this guide was in *Nelson Lagoon* where, for nine years, I was the grades 7-12 teacher (all subject areas) and also building administrator: called the principal-teacher most years and the head teacher some years. It was a component of my "Interdisciplinary Study Guide, the World According to Mr. Grimes." Like other subject area ISG components, it was initially just a collection of handouts and study sheets that I had developed, but I continued to organize it while teaching 20 plus years in *Alaska* and Arizona. The guide improved when I taught math exclusively 1999-2002 in the Indian Oasis School-Baboquivari Unified School District, Tohono O'odham Nation near Sells, Arizona. I also taught in Sells 1976-1980 before teaching in Alaska: usually science exclusively.